A-031 **A CAUSALITY BASED IMAGING METHOD**

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Summary

An imaging condition is derived based on the causality principle. This condition determines directly the velocity contrast over an interface, using the wavefields measured at this interface. We assume the velocity below the interface is constant in vertical (but not necessarily in lateral) direction for a thin layer. Using the reciprocity theorem, we can now calculate the wavefield below this thin layer in the case it was removed and replaced by a layer with the background velocity. If the imaging condition is applied to this field, we can calculate the velocity of the next thin layer. The imaging condition proves to give very good results for 1D and 2D-horizontally layered media.

Introduction

When an imaging procedure is to be satisfactorily applied to 4D-seismic exploration it is required to be both very precise and able to deal with laterally variant media. So far, accurate imaging procedures based on the layer-stripping method have been derived for laterally homogeneous media only, like Yagle and Levy [1]. A technique for inhomogeneous media was proposed by Fokkema et al. [2]. This technique requires a background velocity model. The method proposed here uses the same velocity replacement techniques as shown in [2], but an imaging condition is introduced such that a background velocity model is no longer necessary. First, the derivation of the causality-based imaging condition is shown. After this a short explanation of the wavefield extrapolation and velocity replacement is given. Some results are shown for the simplified horizontally layered case.

Derivation of imaging condition

The imaging condition is used to determine the contrast over the interface between the homogeneous background medium and a thin, laterally varying layer which is about to be stripped. For the derivation of the

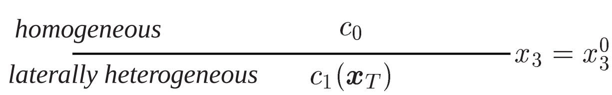

Figure 1: Configuration for derivation of imaging condition.

imaging condition we assume a configuration as shown in Fig. 2, where the upper half-space is homogeneous, and in the lower half-space the wavespeed is variant in the lateral direction only. This means that below the interface there are only downgoing waves. We assume no contrast in mass density. Source and receiver are positioned on top of the interface. The two boundary conditions on the interface are continuity of the pressure wavefield and continuity of the particle velocity, the latter being expressed by:

$$\partial_3^{\uparrow} P\big|_{x_3^0} = \partial_3^{\downarrow} P\big|_{x_3^0}. \tag{1}$$

Since the $\boldsymbol{i}_3$-direction is downwards, $\partial_3^{\uparrow}$ represents the derivative in the homogeneous part of the medium, just above the interface, and $\partial_3^{\downarrow}$ represents the derivative just below the interface. The total pressure wavefield P can be decomposed into an incident and reflected part as follows:

$$\bar{P} = \bar{P}^i + \bar{P}^r. \tag{2}$$

The overbar indicates that the fields are transformed to the spatial Fourier domain. Due to the difference in propagation of the downgoing incident and upgoing reflected wavefield we can write for the area just above the interface:

$$\partial_3^{\uparrow} \bar{P}\big|_{x_3^0} = s\Gamma_0(\bar{P}^r - \bar{P}^i). \tag{3}$$

In this equation $s = j\omega$ is the Laplace parameter, Γ_0 is the vertical slowness, described by $\Gamma_0 = (\frac{1}{c_0^2} + \boldsymbol{\alpha}_T \cdot \boldsymbol{\alpha}_T)^{1/2}$ where $\boldsymbol{\alpha}_T$ is the horizontal slowness. In Fokkema et al. [2] it was shown that the second derivative of the pressure wavefield shows a jump over the interface that is proportional to the contrast over this interface:

$$\{(\partial_3^{\uparrow})^2 - (\partial_3^{\downarrow})^2\}\bar{P} = s^2\mathcal{K}\bar{P}. \tag{4}$$

The operation $\mathcal{K}\bar{P}$ is a compact way of writing the convolution in the transformed domain:

$$\begin{aligned} &\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{P}(js\boldsymbol{\alpha}_T, x_3|\boldsymbol{x}^S) = \\ &\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}'_T\epsilon\mathbb{R}^2} \bar{K}(js\boldsymbol{\alpha}_T - js\boldsymbol{\alpha}'_T)\bar{P}(js\boldsymbol{\alpha}'_T, x_3|\boldsymbol{x}^S)\mathrm{dA}, \end{aligned} \tag{5}$$

where $\bar{K}$ is the spatial Fourier transformation of the contrast term K, which is given by $K = \frac{1}{c_0^2} - \frac{1}{c_1^2(\boldsymbol{x}_T)}$. Eq. (4) can be rewritten as:

$$(\partial_3^{\downarrow})^2\bar{P} = (\partial_3^{\uparrow})^2\bar{P} - s^2\mathcal{K}\bar{P}, \tag{6}$$

and using $(\partial_3^{\uparrow})^2\bar{P} = s^2\Gamma_0^2\bar{P}$ as:

$$\begin{aligned} (\partial_3^{\downarrow})^2\bar{P} &= s^2(\Gamma_0^2 - \mathcal{K})\bar{P}, \\ (\partial_3^{\downarrow})\bar{P} &= -s\sqrt{\Gamma_0^2 - \mathcal{K}}\bar{P}. \end{aligned} \tag{7}$$

Note that the term $\sqrt{\Gamma_0^2 - \mathcal{K}}$ is a pseudo-differential operator.

Using the continuity of the particle velocity, Eq. (1), and substituting Eq. (3):

$$\Gamma_0(\bar{P}^r - \bar{P}^i) = -\sqrt{\Gamma_0^2 - \mathcal{K}}\bar{P}. \tag{8}$$

Now multiply Eq. (2) by Γ_0:

$$\Gamma_0\bar{P}^r + \Gamma_0\bar{P}^i = \Gamma_0\bar{P}, \tag{9}$$

and add the last two equations:

$$2\Gamma_0\bar{P}^r = (-\sqrt{\Gamma_0^2 - \mathcal{K}} + \Gamma_0)\bar{P}. \tag{10}$$

Subtracting the same two equations results in:

$$2\Gamma_0\bar{P}^i = (\sqrt{\Gamma_0^2 - \mathcal{K}} + \Gamma_0)\bar{P}. \tag{11}$$

The inner product is defined as:

$$\langle \bar{f}, \bar{g} \rangle = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} \bar{f}(\boldsymbol{\alpha}_T)\bar{g}(-\boldsymbol{\alpha}_T)\mathrm{d}\boldsymbol{\alpha}_T. \tag{12}$$

Now, using $\langle \bar{f}, \bar{g} \rangle = \langle \bar{g}, \bar{f} \rangle$, we obtain: $4\langle \Gamma_0 \bar{P}^r, \Gamma_0 \bar{P}^i \rangle = \langle \bar{P}, \mathcal{K}\bar{P} \rangle$, where we used the fact that the pseudo-differential operator $\sqrt{\Gamma_0^2 - \mathcal{K}}$ is a symmetric operator, which follows from the symmetry of the square-root operator in the space domain, see [3]. Now we return to a configuration with a lower halfspace that is heterogeneous in both vertical and horizontal direction. This means that the wavefield below the interface consists of both up- and downgoing waves. Due to causality, the upgoing part will always need a lapse of time to arrive at the interface. For this reason the imaging condition derived above is still valid, but only at times very close to $t = 0$. This also involves having to analyze each interface separately at time $t = 0$, so the entire imaging procedure has a recursive character. We now write for the imaging condition:

$$\mathcal{F}^{-1}[4\langle \Gamma_0 \bar{P}^r, \Gamma_0 \bar{P}^i \rangle] = \mathcal{F}^{-1}[\langle \bar{P}, \mathcal{K}\bar{P} \rangle], \quad t = 0, \tag{13}$$

where $\mathcal{F}^{-1}$ stands for the inverse temporal Fourier transform. The inner products are calculated in the frequency domain, after which they are transformed back to the time domain, and the contrast operator $\mathcal{K}$ is determined for time $t = 0$.

Wavefield extrapolation and velocity replacement

We use the velocity replacement techniques as explained by Fokkema et al. in [2]. The theory is based on the reciprocity theorem, see [4]. This reciprocity theorem is applied to two states as shown in figure 2. The subsurface is derived in thin horizontal layers. These thin layers are assumed not to vary in vertical direction, but can be heterogeneous in lateral direction. The first state is the actual configuration and in the second state a thin horizontal layer is removed from the top and replaced by a layer with the same velocity as the background medium.

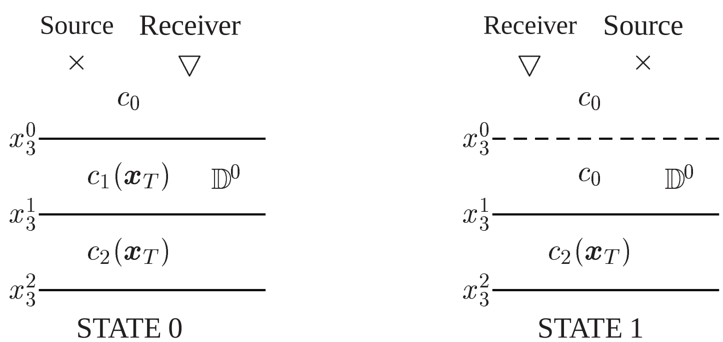

Figure 2: Configuration of the two states in Rayleigh's reciprocity theorem.

Using the expressions for the incident and reflected fields and choosing the right positions for the receiver,see [3], an expression can be derived with which the total wavefield in state 0 just below the top layer, at level x_3^1, can be determined. This expression can be solved if we know the contrast over the interface, which is found using the imaging condition. The next step is to calculate the wavefield at x_3^1 in case the top layer was replaced by a layer with the velocity of the homogeneous background medium (in state 1). This situation now becomes the new state 0, and using the imaging condition we can determine the velocity contrast over the next layer. This procedure can be repeated until the desired velocity profile of the subsurface is found. The application of the imaging condition, which directly determines the contrast over an interface, is what makes this method different from other imaging procedures.

Some results

To test the algorithm, some experiments were done for a one-dimensional medium with two interfaces, one at a depth of 150 m and one at a depth of 300 m. The velocity of the background medium is 1500 m/s, of the first layer 2500 m/s, and of the second layer 6000 m/s. The velocity model is shown in Figure 3, together with the velocities determined by the layer-stripping procedure. The thickness of the stripped layers is 1.5 m. For the wavelet a zero-phase Gaussian was used. Instead of using just the value of the wavefields at time $t = 0$ when applying the imaging condition, the energy of the wavelet around time $t = 0$ was used. The results are very close to the modeled velocities. Figure 3 shows the calculated upgoing field in state 1 after every 20 layer-stripping steps. The upper trace is the syntheticly generated trace. The internal multiple is visible at about 0.63s. Obviously, the primary reflection moves closer to time $t = 0$ when the medium is penetrated deeper. When the first reflection reaches time $t = 0$ it is 'detected' by the algorithm, and the contrast gets a value while the second reflection keeps moving to the $t = 0$-axis, now with a different speed. The multiple reflection now starts travelling with double speed towards time $t = 0$ and reaches this point exactly together with the primary reflection that caused it. This is why the multiple does not cause a (wrong) change in the calculated velocities, as it would have if it were handled as a separate event.

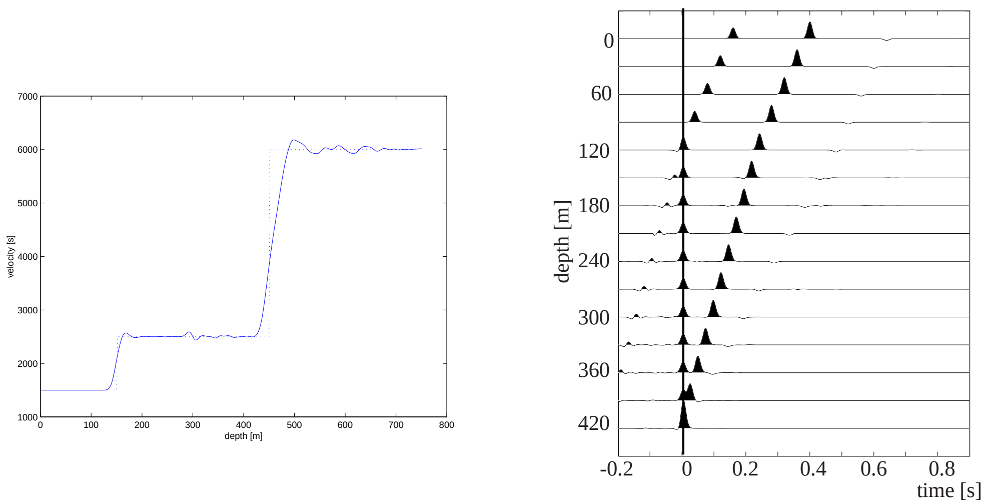

Figure 3: On the left: Three layer velocity model (dashed) together with the calculated velocities (solid). On the right: Calculated reflected field in state 1 for every 20 layer-stripping steps.

Conclusions

The theory of an imaging condition which is based on the causality principle is derived. This imaging condition yields directly the contrast parameters of the medium, layer by layer. The imaging condition is used together with a layer replacement method based on the reciprocity theorem. This method makes sure the internal multiples in the data are dealt with correctly. Testing for horizontally layered media shows good results. The laterally varying case involves solving convolutional and pseudo-differential operators. This is what our current work focuses on, as well as on the application of this algorithm to time-lapse problems.

References

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