Homogenization through velocity replacement in inhomogeneous acoustic media

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Introduction. In the following analysis we consider the action of a point source of the injection type in an acoustic medium. The seismic medium response is recorded with a set of point receivers. We assume that the seismic preprocessing has resulted in a configuration model whereby the semi-infinite upper half-space is homogeneous in nature characterized by the acoustic wavespeed c_0. In this halfspace the point sources and receivers are positioned. The lower semi-infinite halfspace is our domain of interest and is occupied with an inhomogeneous medium characterized by the wavespeed $c(\mathbf{x})$. We further assume that there is no contrast in mass density and hence is constant throughout our whole configuration. In this paper it is our aim starting from the original configuration to successively replace the inhomogeneous lower halfspace in small steps with the homogeneous background material (velocity replacement) and subsequently determine the resulting wavefield in the modified configuration.

Contrast formulation. Our starting point is Rayleigh's reciprocity theorem formulated in the space-frequency domain with frequency parameter $s = j\omega$ (Fokkema and van den Berg, 1993, Seismic applications of acoustic reciprocity, Elsevier). The spatial domain of consideration is depicted in Figure 1. The left-hand side shows the starting configuration, which is denoted by state 0. The inhomogeneous medium is divided in the x_3-direction in thin slabs of width $\Delta x_3 = x_3^1 - x_3^0$ in which the wavespeed has only variation in the lateral direction. We indicate the wavespeed in the first layer with $c_1(\mathbf{x}_T)$, where $\mathbf{x}_T$ is the transverse position vector in the x_1, x_2-direction. On the right-hand side of Figure 1 we show the situation in state 1 where the inhomogeneous wavespeed $c_1(\mathbf{x}_T)$ in the first layer has been replaced by the wavespeed of the upper halfspace c_0. Then application of the reciprocity theorem to $\mathbb{R}^3$, considering the states 0 and 1, and reversing the positions of the sources and receivers in state 1 with respect to state 0 leads to the following interaction relation between the wave fields $\hat{P}^0$ in state 0 and $\hat{P}^1$ in state 1

$$\int_{\mathbf{x}\in\mathbb{D}^0} s^2 K(\mathbf{x}_T)\hat{P}^0(\mathbf{x}|\mathbf{x}^S)\hat{P}^1(\mathbf{x}|\mathbf{x}^R)\mathrm{d}V = \hat{W}\left[\hat{P}^0(\mathbf{x}^R|\mathbf{x}^S) - \hat{P}^1(\mathbf{x}^S|\mathbf{x}^R)\right], \tag{1}$$

where we have omitted the explicit dependence on s in the wavefields. $\hat{W}$ denotes the spectrum of the source wavelet and the contrast function K is given by $K(\mathbf{x}_T) = c_0^{-2} - c_1^{-2}(\mathbf{x}_T)$. Then application of the spatial Fourier transformation with respect to the horizontal receiver and source coordinates and exponential transformation kernels $\exp(js\boldsymbol{\alpha}_T^R \cdot \mathbf{x}_T^R)$ and $\exp(-js\boldsymbol{\alpha}_T^S \cdot \mathbf{x}_T^S)$ leads to the following integral equation equivalent of equation (1)

$$\begin{aligned}\frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T\in\mathbb{R}^2}\mathrm{d}A\int_{x_3^0}^{x_3^1}\bar{\bar{P}}^1(-js\boldsymbol{\alpha}_T, x_3|js\boldsymbol{\alpha}_T^R, x_3^R)s^2\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T, x_3|-js\boldsymbol{\alpha}_T^S, x_3^S)\mathrm{d}x_3 \\ = \hat{W}\left[\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^R|-js\boldsymbol{\alpha}_T^S, x_3^S) - \bar{\bar{P}}^1(-js\boldsymbol{\alpha}_T^S, x_3^S|js\boldsymbol{\alpha}_T^R, x_3^R)\right],\end{aligned} \tag{2}$$

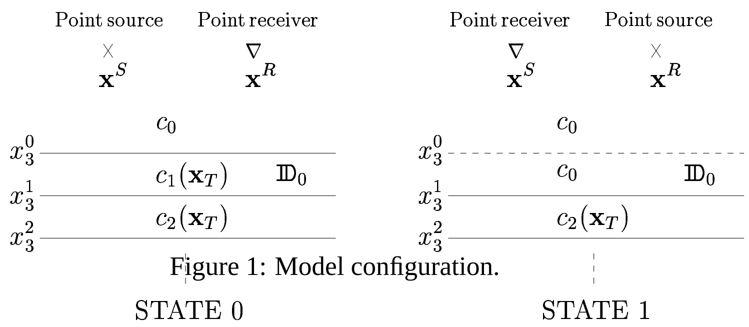

Figure 1: Model configuration.

where we also used Parseval's theorem. The convolution operator $\mathcal{K}$ operating on $\bar{\bar{P}}^0$ is defined through

$$\mathcal{K}(js\boldsymbol{\alpha}_T)\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S) = \frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}'_T\in\mathbb{R}^2} \bar{K}(js\boldsymbol{\alpha}_T - js\boldsymbol{\alpha}'_T)\bar{\bar{P}}^0(js\boldsymbol{\alpha}'_T, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S)\mathrm{d}A. \tag{3}$$

Wavefield extrapolation. Next we evaluate equation (2) at two levels, viz. $x_3 = x_3^0$ and $x_3 = x_3^1$. The resulting two expressions can be used to eliminate $\bar{\bar{P}}^1$ and leads to the following extrapolation result for $\bar{\bar{P}}^0$ in the thin slab

$$\begin{aligned}&\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3| - js\boldsymbol{\alpha}_T^S, x_3^S) = \\ &\left[\cosh(s\mathcal{F}(js\boldsymbol{\alpha}_T^R)(x_3 - x_3^0)) + \frac{\sinh(s\mathcal{F}(js\boldsymbol{\alpha}_T^R)(x_3 - x_3^0))}{s\mathcal{F}(js\boldsymbol{\alpha}_T^R)}\partial_3^{\uparrow}\right]\bar{\bar{P}}^0(js\boldsymbol{\alpha}_T^R, x_3^0| - js\boldsymbol{\alpha}_T^S, x_3^S),\end{aligned} \tag{4}$$

where $\mathcal{F}$ is the square-root operator, defined as

$$\mathcal{F}(js\boldsymbol{\alpha}_T^R) = \sqrt{(\Gamma_0^R)^2 - \mathcal{K}(js\boldsymbol{\alpha}_T^R)}. \tag{5}$$

In fact $\mathcal{F}$ constitutes a pseudo-differential operator in the transformed domain. The result of equation (4) has been achieved through consistent use of Taylor expansions. In our analysis we obtain the additional result:

$$(\partial_3^{\uparrow})^0\bar{\bar{P}}^0 = (\partial_3^{\downarrow})^0\bar{\bar{P}}^0, \quad (\partial_3^{\uparrow})^1\bar{\bar{P}}^0 = (\partial_3^{\downarrow})^1\bar{\bar{P}}^0, \quad \{(\partial_3^{\uparrow})^2 - s^2\mathcal{K}\}\bar{\bar{P}}^0 = (\partial_3^{\downarrow})^2\bar{\bar{P}}^0. \tag{6}$$

The operators $\partial_3^{\uparrow}$ and $\partial_3^{\downarrow}$ denote the normal derivatives for $x_3 \uparrow x_3^0$ and $x_3 \downarrow x_3^0$, respectively. The first identity learns that at the discontinuity boundary x_3^0 the pressure wavefield is continuous. This is not a new result, but the standard boundary condition. Because the mass density does not vary in our configuration, the second identity represents the continuity of the particle velocity at the discontinuity level. Finally the third identity states that the second normal partial derivative of the pressure wavefield jumps across the level of the discontinuity to an amount that corresponds to the contrast in wavespeeds under the assumption that the mass density does not vary.

Velocity replacement. In our further analysis we substitute the result of equation (4) into equation (3), leading to the following integral equation of the second kind for $\bar{\bar{P}}^1$ (symbolized by X) at the level x_3^0

$$X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) = Y(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T^S) - \frac{1}{(2\pi)^2}\int_{s\boldsymbol{\alpha}_T\in\mathbb{R}^2} L(js\boldsymbol{\alpha}_T, js\boldsymbol{\alpha}_T^S)X(js\boldsymbol{\alpha}_T^R, js\boldsymbol{\alpha}_T)\mathrm{d}A. \tag{7}$$

In this integral equation the known term Y and the kernel function L are related to the result of pseudo-differential operations on $\bar{\bar{P}}^0$ and $\bar{\bar{P}}^0/\hat{W}$, respectively. We observe that equation (7) has a similar form as the operational expression that is used in the removal of surface-related multiples for marine seismic data [Verschuur et al., 1992, Adaptive surface-related multiple elimination, Geophysics **57**, 1166-1177; Fokkema and van den Berg, 1993, Elsevier; van Borselen, 1995, Removal of surface-related multiples from marine seismic data, Ph.D. thesis, Delft Univ. of Techn.; van Borselen et al., 1996, Removal of surface-related wave phenomena, the marine case, Geophysics **61**, 202-210]. To solve this integral equation we also use in this case a Neumann iterative solution. From the correspondence with the solution of the surface-related multiple problem we conclude that the iterative terms in our solution constitute the removal of the internal multiples in the slab from the data in state 0. It is obvious that the whole process can be used in a recursive fashion to homogenize the inhomogeneous acoustic medium. Furthermore, the kernel has to be deconvolved for the wavelet. This allows us to estimate the wavelet after every recursion step using the minimum energy norm. This re-estimation of the wavelet could be profitable to stabilize our replacement process.

Conclusions. In this paper we have shown how Rayleigh's reciprocity theorem aids the velocity replacement in thin slabs of an inhomogeneous acoustic medium. This process is divided into two steps: first the extrapolation of the original wavefield into the inhomogeneous thin slab is carried out (equation 4). The resulting expression is formulated in terms of pseudo-differential operators that act on the original wavefield. The second step encompasses the actual removal of the influence of the thin slab on the original wavefield. This operation is furnished by an integral equation of the second kind (equation 7) and is in nature closely related to the process of surface-related multiple removal in marine seismic data processing. There the solution of the removal process is achieved by an iterative Neumann series expansion. Also in the case of velocity replacement we propose to use such an approach. In a similar way the successive terms in the velocity replacement Neumann series correspond with the internal multiples in the thin slab. We have observed that the procedure can be done in an iterative fashion so that we can successively peel off the inhomogeneous medium and in doing so replace it with the homogeneous background. Our future work will focus on the numerical stabilization of the process and application of the result in seismic imaging techniques.