Modeling, imaging and characterization of self-similar reflectors

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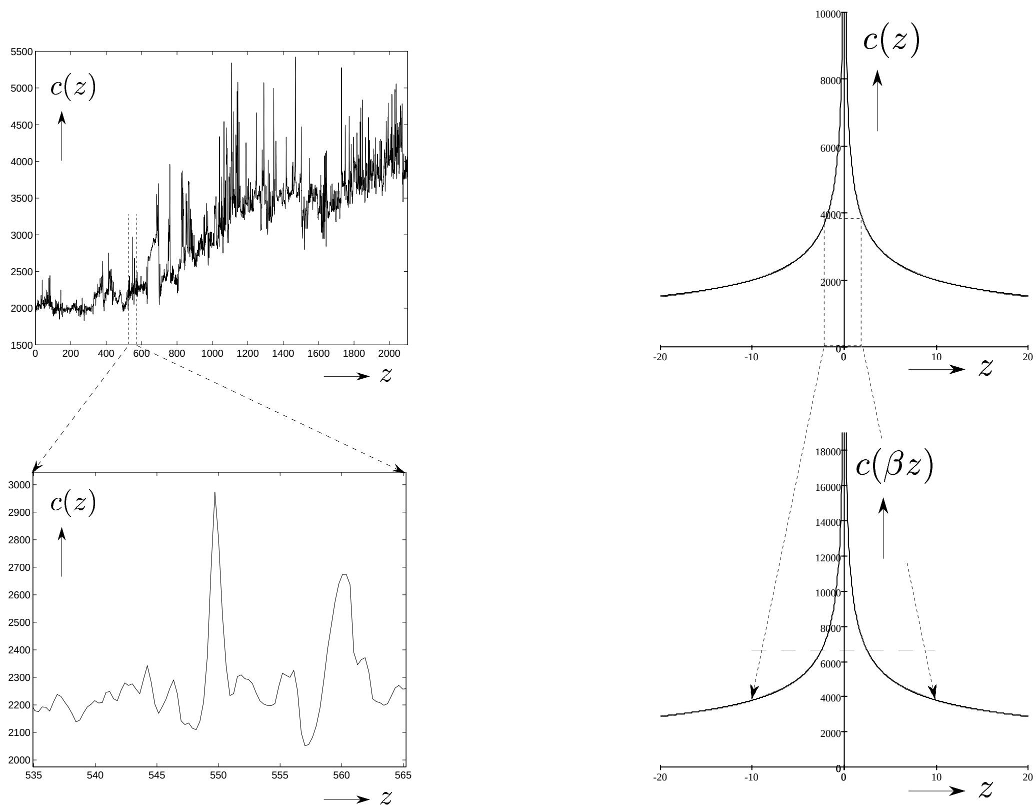
Introduction. Amplitude-versus-angle (AVA) analysis is generally based on a model consisting of two homogeneous layers, separated by a horizontal interface. This implies that the medium parameters are assumed to behave as step-functions of the depth coordinate z, at least in a finite region around the interface. However, looking at well-logs of, for example, the compressional wave velocity $c(z)$, it appears that the main outliers, responsible for the main reflections, behave quite different from step-functions, see Figure 1a. In this paper we represent reflectors by functions of the form $c(z) = c_1\,|z/z_1|^\alpha$. This function nicely captures the singular behaviour of the type of outlier, observed in Figure 1a at $z = 550$ m. Moreover, this function is *self-similar*, according to $c(\beta z) = \beta^\alpha c(z)$, for $\beta > 0$, see Figure 1b.

Forward model. It appears that the reflection coefficient R of self-similar reflectors is a function of angle and frequency, or, equivalently, of rayparameter and scale: $R = R(\mathrm{p}, \sigma)$. Moreover, this function appears to be self-similar as well, according to $R(\mathrm{p}, \sigma) = R(\beta^\alpha \mathrm{p}, \beta^{\alpha-1}\sigma)$, meaning that $R(\mathrm{p}, \sigma)$ is constant on curves described by $\mathrm{p}^{1-\alpha}\sigma^\alpha = \mathrm{constant}$, see Figure 2. Note that these curves depend on the scaling coefficient α.

Imaging. At last years EAGE we presented a new AVA imaging method that accounts for the propagation and reflection related effects of fine-layering (i.e., for dispersion and interference, respectively). For the well-log of Figure 1a the results of modeling and AVA imaging are shown in Figure 3.



(a) *(b)*

Figure 1: (a) Well-log of P-wave velocity and a close-up of an outlier. (b) The self-similar function $c(z) = c_1\,|z/z_1|^\alpha$.

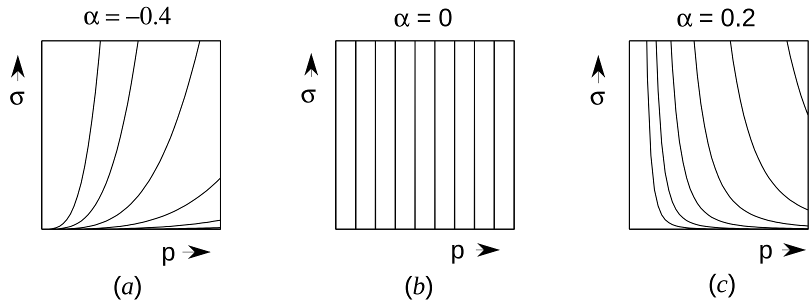


Figure 2: Curves, along which the reflection coefficient $R(\mathrm{p}, \sigma)$ is constant for $\alpha = -0.4$, 0 and 0.2, respectively.

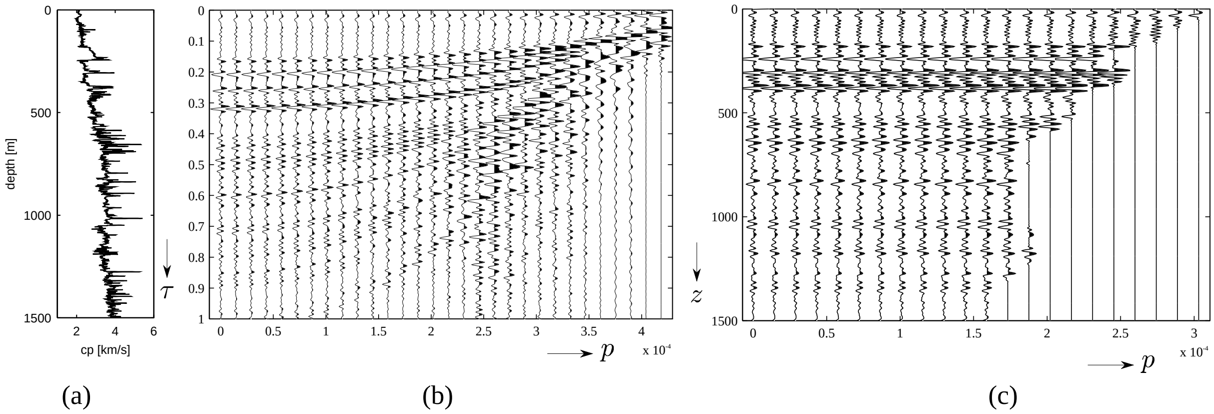


Figure 3: (a) Well-log. (b) Modeling result in τ, p-domain. (c) AVA imaging result in z, p-domain.

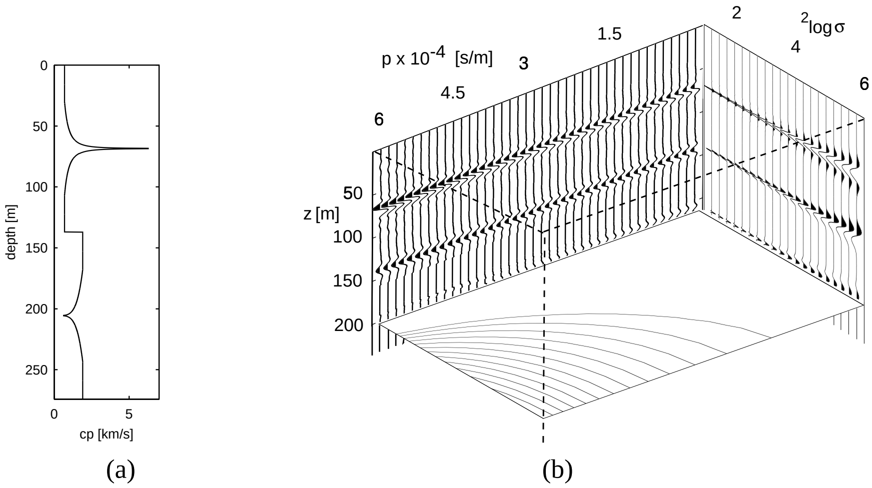


Figure 4: (a) Well-log. (b) Wavelet transformed AVA image.

Characterization. For the characterization of the singularities in the imaged section we propose a procedure based on the wavelet transform. The method will be outlined with the example in Figure 4. Figure 4a shows a simplified well-log that contains three singularities, with $\alpha = -0.4$, 0 and 0.2, respectively. The left back-plane in Figure 4b shows the AVA imaging result in the z, p-domain, analogous to Figure 3c. By applying the wavelet transform (along the z-axis) we obtain a 3-D data cube, containing $R(z, \mathrm{p}, \sigma)$. The right back-plane in Figure 4b shows $R(z, \mathrm{p} = 0, \sigma)$. The horizontal cross-section at $z = 210$m shows contours of constant $R(z = 210, \mathrm{p}, \sigma)$. Note that these contours accurately resemble those in Figure 2c, hence, from this analysis we conclude that α equals approximately 0.2 for the imaged singularity at $z = 210$, as expected. A similar analysis of the other two singularities yielded accurate estimates of the expected values $\alpha = -0.4$ and $\alpha = 0$, respectively.

Conclusions. We have shown that the reflection coefficient of self-similar reflectors (Figure 1) is self-similar as well (Figure 2). Moreover, we showed that the singularity exponents α can be obtained from the seismic data by applying a wavelet transform to the AVA imaging result in the z, p-domain (Figure 4). The exponent α may prove to be a useful indicator in seismic characterization.