Decomposition of one-way representations and one-way operators

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September 28, 1995

One-way representation in integral form

One-way representations of seismic reflection data are the "forward models" for prestack depth migration schemes. We present an overview of various decompositions of one-way representations as well as of one-way operators.

In the space-frequency domain the one-way representation of seismic reflection data reads

$$P^-(\mathbf{x}_D, \mathbf{x}_S) = \int_{\mathcal{V}} W^-(\mathbf{x}_D, \mathbf{x})\hat{R}^+(\mathbf{x})W^+(\mathbf{x}, \mathbf{x}_S)S_0^+(\mathbf{x}_S)d^3\mathbf{x}. \tag{1}$$

The angular frequency ω is suppressed for notational convenience. From right to left, one encounters subsequently: downward propagation from the source at $\mathbf{x}_S$ to $\mathbf{x}$, reflection at $\mathbf{x}$ and upward propagation from $\mathbf{x}$ to the detector at $\mathbf{x}_D$. At the last EAEG meeting the first author has shown that this representation applies for primary as well as "generalized primary" reflections. In the latter case the extrapolators W^+ and W^- account for the angle dependent dispersion effects related to internal multiple scattering between the small scale heterogeneities.

One-way representation in matrix form

In Berkhout's matrix notation the one-way representation of equation (1) reads

$$\underset{\sim}{\mathbf{P}}^-(z_D, z_S) = \sum_{n=1}^{\infty} \underset{\sim}{\mathbf{W}}^-(z_D, z_n)\underset{\sim}{\mathbf{R}}^+(z_n)\underset{\sim}{\mathbf{W}}^+(z_n, z_S)\underset{\sim}{\mathbf{S}}_0^+(z_S). \tag{2}$$

Each column of matrix $\underset{\sim}{\mathbf{P}}^-(z_D, z_S)$ represents a discretized shot record. The pth element in the qth column denotes the response at the pth detector related to the qth source. This is visualized in Figure 1a for $z_D = z_S$. The other matrices in equation (2) are defined in a similar way. Matrix $\underset{\sim}{\mathbf{W}}^+(z_n, z_S)$ is shown in Figure 2a for the special case of a homogeneous macro model. Prestack depth migration essentially comes to resolving $\underset{\sim}{\mathbf{R}}^+(z_n)$ from equation (2) by appling the inverse versions of matrices $\underset{\sim}{\mathbf{W}}^+$ and $\underset{\sim}{\mathbf{W}}^-$ to the data matrix $\underset{\sim}{\mathbf{P}}^-(z_D, z_S)$. When the aforementioned dispersion effects are negligible, the inverse matrices may be approximated by $\{\underset{\sim}{\mathbf{W}}^+\}^H$ and $\{\underset{\sim}{\mathbf{W}}^-\}^H$, respectively, where ^{H} denotes the complex conjugate transpose.

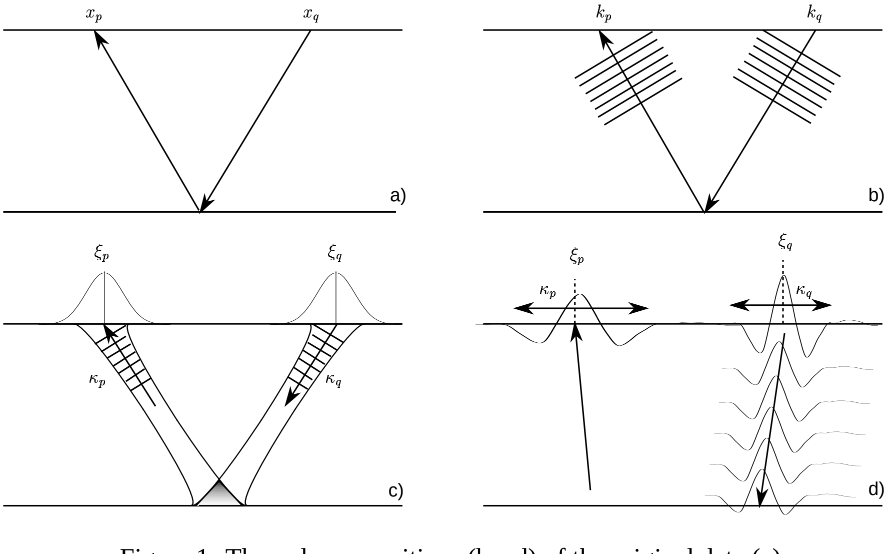

Figure 1: Three decompositions (b,c,d) of the original data (a).

Decomposition of the data matrix

We apply a transformation to the data matrix $\mathbf{P}^-(z_D, z_S)$, according to

$$\tilde{\mathbf{P}}^-(z_D, z_S) = \mathbf{\Gamma}\mathbf{P}^-(z_D, z_S)\mathbf{\Gamma}^{-1}. \tag{3}$$

The transform matrix $\mathbf{\Gamma}$ may stand for the Fourier transform, the Gabor transform or for the wavelet transform. In these three cases, equation (3) represents a decomposition in "plane wave experiments" (Figure 1b), "Gaussian beam experiments" (Figure 1c), or "scale experiments" (Figure 1d), respectively.

Decomposition of the one-way representation

The representation for the transformed data matrix $\tilde{\mathbf{P}}^-(z_D, z_S)$ follows easily by applying the transform matrices $\mathbf{\Gamma}$ and $\mathbf{\Gamma}^{-1}$ to equation (2) in the following manner

$$\underbrace{\mathbf{\Gamma}\mathbf{P}^-\mathbf{\Gamma}^{-1}}_{\tilde{\mathbf{P}}^-(z_D,z_S)} = \sum_{n=1}^{\infty} \underbrace{\mathbf{\Gamma}\mathbf{W}^-\mathbf{\Gamma}^{-1}}_{\tilde{\mathbf{W}}^-(z_D,z_n)} \underbrace{\mathbf{\Gamma}\mathbf{R}^+\mathbf{\Gamma}^{-1}}_{\tilde{\mathbf{R}}^+(z_n)} \underbrace{\mathbf{\Gamma}\mathbf{W}^+\mathbf{\Gamma}^{-1}}_{\tilde{\mathbf{W}}^+(z_n,z_S)} \underbrace{\mathbf{\Gamma}\mathbf{S}_0^+\mathbf{\Gamma}^{-1}}_{\tilde{\mathbf{S}}_0^+(z_S)}. \tag{4}$$

The resulting representation has the same form as equation (2). Hence, prestack depth migration of the decomposed data matrices is essentially the same as described below equation (2).

Decomposition of the one-way operators

According to equation (4), decomposition of the one-way downward propagator is described by

$$\tilde{\mathbf{W}}^+(z_n, z_S) = \mathbf{\Gamma}\mathbf{W}^+(z_n, z_S)\mathbf{\Gamma}^{-1}. \tag{5}$$

A similar relation holds for the upward propagator $\mathbf{W}^-(z_D, z_n)$. For the special case of a homogeneous macro model, the decomposed matrix $\tilde{\mathbf{W}}^+(z_n, z_S)$ is shown in Figures 2b,c and d for each of the three interpretations of the transform matrix $\mathbf{\Gamma}$ (Fourier, Gabor and wavelet transform, respectively). Note that any of these matrices is sparser than the original matrix in Figure 2a. In particular, note that the Fourier transformed propagator in Figure 2b is purely diagonal. Hence, for a homogeneous macro model plane wave decomposition is fully equivalent with the eigenvalue decomposition. For inhomogeneous macro models the decomposed matrices are generally less sparse than in Figures 2b,c and d. Of course in this case the eigenvalue decomposition would still yield a diagonal matrix. The theoretical and practical aspects of the eigenvalue decomposition are further discussed in a companion paper [Grimbergen, Wapenaar and Dessing: One-way operators in laterally varying media].

Conclusions

The Fourier transform, the Gabor transform and the wavelet transform of one-way representations and one-way operators have been interpreted in terms of decompositions in plane wave experiments, Gaussian beam experiments and scale experiments. The consequences for prestack depth migration will be discussed during the presentation.

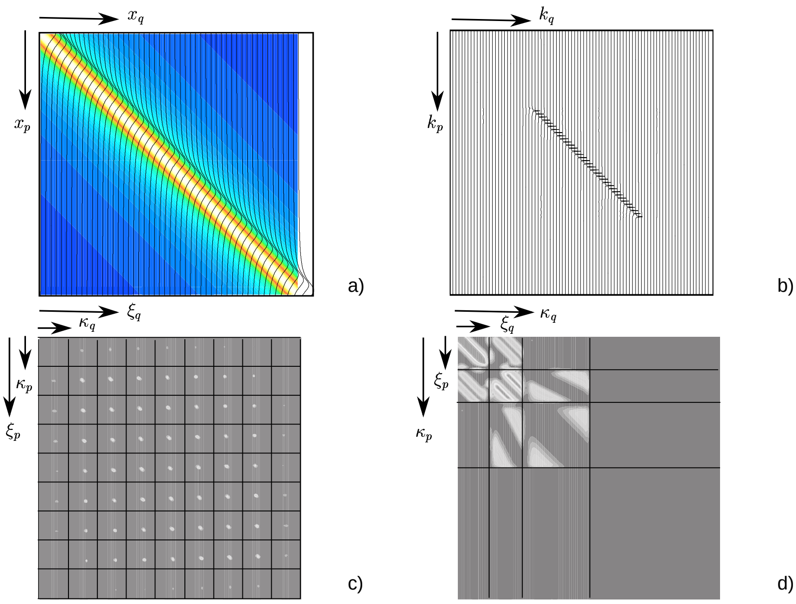

Figure 2: Three decompositions (b,c,d) of the original propagator (a)