Z-99 CODAS IN REFLECTION AND TRANSMISSION RESPONSES AND THEIR MUTUAL RELATIONS

KEES WAPENAAR, DEYAN DRAGANOV and JACOB FOKKEMA
Section of Applied Geophysics, Centre for Technical Geoscience, Delft University of Technology, The Netherlands

Introduction

Codas due to internal multiple scattering contain relevant information about the heterogeneities in the subsurface. Employing the information in the coda in seismic imaging may improve the resolution. Codas occur in seismic reflection as well as in transmission data. In this paper we derive two relations between reflection and transmission responses in 3-D inhomogeneous media. Based on these relations, we will show that the reflection response, includings its coda, can be derived from the transmission response and, vice versa, that the coda of the transmission response can be derived from the reflection response. The former relationship was derived for horizontally layered media by Claerbout (Geophysics, 1968) and the latter by Herman (Inverse Problems, 1992) and Wapenaar and Herrmann (SEG, 1993). In the current paper we employ a uniform approach for deriving both relationships, using reciprocity; both results are valid for 3-D inhomogeneous media. We also indicate how these results may be of use for seismic imaging.

One-way reciprocity theorem

We use acoustic reciprocity as a starting point to derive the relations between the reflection and transmission responses. Acoustic reciprocity formulates a relation between two acoustic states in one and the same domain (de Hoop, J. Acoust. Soc. Am., 1988; Fokkema and van den Berg, Elsevier, 1993). The two states will be distinguished by subscripts A and B. Usually reciprocity theorems apply to the full wave fields in both states. As an alternative, Wapenaar and Grimbergen (Geoph. J. Int., 1996) derived reciprocity theorems for one-way (i.e. downgoing and upgoing) wave fields. For a plan-parallel domain $\mathcal{D}$ embedded between surfaces $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$ at depth levels $x_{3,0}+\epsilon$ and $x_{3,m}$, respectively, the one-way reciprocity theorem of the correlation type reads in the space-frequency $(\mathbf{x},\omega)$ domain

$$\int_{\partial\mathcal{D}_0}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}\mathrm{d}^2\mathbf{x}_H = \int_{\partial\mathcal{D}_m}\{(P_A^+)^*P_B^+ - (P_A^-)^*P_B^-\}\mathrm{d}^2\mathbf{x}_H, \quad (1)$$

where P^+ and P^- are flux-normalized downgoing and upgoing wave fields, respectively, * denotes complex conjugation and $\mathbf{x}_H=(x_1,x_2)$. In equation (1) it has been assumed that the medium parameters in both states are identical, lossless and 3-D inhomogeneous and that the domain $\mathcal{D}$ is source-free.

Relations between reflection and transmission responses

We employ equation (1) to derive relations between the reflection and transmission responses of the medium in domain $\mathcal{D}$. To this end we consider two configurations. In the first configuration (Figure 1a) *both* half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$ are homogeneous. For states A and B we choose sources for downgoing waves at $\mathbf{x}_A$ and $\mathbf{x}_B$ in the upper half-space, just above $\partial\mathcal{D}_0$, that is, we define $\mathbf{x}_A=(\mathbf{x}_{H,A},x_{3,0})$ and $\mathbf{x}_B=(\mathbf{x}_{H,B},x_{3,0})$. Hence, for $\mathbf{x}$ at $\partial\mathcal{D}_0$ we have

$$P_A^+(\mathbf{x},\mathbf{x}_A,\omega) = \delta(\mathbf{x}_H-\mathbf{x}_{H,A})s_A(\omega), \quad (2)$$

$$P_B^+(\mathbf{x},\mathbf{x}_B,\omega) = \delta(\mathbf{x}_H-\mathbf{x}_{H,B})s_B(\omega), \quad (3)$$

$$P_A^-(\mathbf{x},\mathbf{x}_A,\omega) = R_0(\mathbf{x},\mathbf{x}_A,\omega)s_A(\omega), \quad (4)$$

$$P_B^-(\mathbf{x},\mathbf{x}_B,\omega) = R_0(\mathbf{x},\mathbf{x}_B,\omega)s_B(\omega), \quad (5)$$

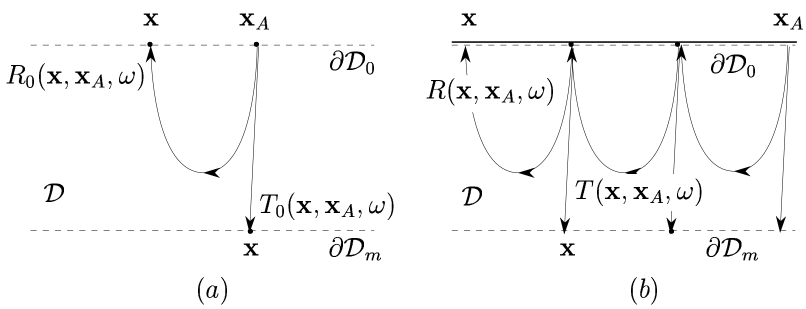


Figure 1: *Domain $\mathcal{D}$ between surfaces $\partial\mathcal{D}_0$ and $\partial\mathcal{D}_m$. The medium in $\mathcal{D}$ is inhomogeneous in the x_1-, x_2- and x_3-directions. In (a) the half-spaces above $\partial\mathcal{D}_0$ and below $\partial\mathcal{D}_m$ are homogeneous. In (b) there is a free surface just above $\partial\mathcal{D}_0$; the half-space below $\partial\mathcal{D}_m$ is again homogeneous.*

where $s_A(\omega)$ and $s_B(\omega)$ are the source spectra for both states. $R_0(\mathbf{x}, \mathbf{x}_A, \omega)$ is the reflection response of the inhomogeneous medium in $\mathcal{D}$, including all internal multiples, for a source at $\mathbf{x}_A$ and a receiver at $\mathbf{x}$ (Figure 1a). A similar remark applies to $R_0(\mathbf{x}, \mathbf{x}_B, \omega)$. The subscript $_0$ denotes that no free surface multiples are included. For $\mathbf{x}$ at $\partial\mathcal{D}_m$ we have

$$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = T_0(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega), \tag{6}$$
$$P_B^+(\mathbf{x}, \mathbf{x}_B, \omega) = T_0(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega), \tag{7}$$
$$P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = P_B^-(\mathbf{x}, \mathbf{x}_B, \omega) = 0, \tag{8}$$

where $T_0(\mathbf{x}, \mathbf{x}_A, \omega)$ is the transmission response of the inhomogeneous medium in $\mathcal{D}$, including all internal multiples (Figure 1a). Substitution into equation (1) and dividing both sides of the equation by $s_A^*(\omega)s_B(\omega)$ yields

$$\boxed{\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - \int_{\partial\mathcal{D}_0} R_0^*(\mathbf{x}, \mathbf{x}_A, \omega) R_0(\mathbf{x}, \mathbf{x}_B, \omega) \mathrm{d}^2\mathbf{x}_H = \int_{\partial\mathcal{D}_m} T_0^*(\mathbf{x}, \mathbf{x}_A, \omega) T_0(\mathbf{x}, \mathbf{x}_B, \omega) \mathrm{d}^2\mathbf{x}_H.} \tag{9}$$

In the second configuration (Figure 1b) we choose a free surface at $x_{3,0}$ and a homogeneous half-space below $\partial\mathcal{D}_m$. The source points $\mathbf{x}_A$ and $\mathbf{x}_B$ are chosen as before. Hence, for $\mathbf{x}$ at $\partial\mathcal{D}_0$ we now have

$$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = \delta(\mathbf{x}_H - \mathbf{x}_{H,A}) s_A(\omega) + r P_A^-(\mathbf{x}, \mathbf{x}_A, \omega), \tag{10}$$
$$P_B^+(\mathbf{x}, \mathbf{x}_B, \omega) = \delta(\mathbf{x}_H - \mathbf{x}_{H,B}) s_B(\omega) + r P_B^-(\mathbf{x}, \mathbf{x}_B, \omega), \tag{11}$$
$$P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = R(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega), \tag{12}$$
$$P_B^-(\mathbf{x}, \mathbf{x}_B, \omega) = R(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega), \tag{13}$$

where r is the reflection coefficient of the free surface ($r = -1$) and $R(\mathbf{x}, \mathbf{x}_A, \omega)$ is the reflection response of the inhomogeneous medium in $\mathcal{D}$, including all internal and free surface multiples (Figure 1b). For $\mathbf{x}$ at $\partial\mathcal{D}_m$ we have

$$P_A^+(\mathbf{x}, \mathbf{x}_A, \omega) = T(\mathbf{x}, \mathbf{x}_A, \omega) s_A(\omega), \tag{14}$$
$$P_B^+(\mathbf{x}, \mathbf{x}_B, \omega) = T(\mathbf{x}, \mathbf{x}_B, \omega) s_B(\omega), \tag{15}$$
$$P_A^-(\mathbf{x}, \mathbf{x}_A, \omega) = P_B^-(\mathbf{x}, \mathbf{x}_B, \omega) = 0, \tag{16}$$

where $T(\mathbf{x}, \mathbf{x}_A, \omega)$ is the transmission response of the inhomogeneous medium in $\mathcal{D}$, including all internal and free surface multiples (Figure 1b). Substitution into equation (1), using $r = -1$ and $R(\mathbf{x}_A, \mathbf{x}_B, \omega) = R(\mathbf{x}_B, \mathbf{x}_A, \omega)$, and dividing both sides of the equation by $s_A^*(\omega)s_B(\omega)$ yields

$$\boxed{\delta(\mathbf{x}_{H,B} - \mathbf{x}_{H,A}) - 2\Re\big[R(\mathbf{x}_A, \mathbf{x}_B, \omega)\big] = \int_{\partial\mathcal{D}_m} T^*(\mathbf{x}, \mathbf{x}_A, \omega) T(\mathbf{x}, \mathbf{x}_B, \omega) \mathrm{d}^2\mathbf{x}_H,} \tag{17}$$

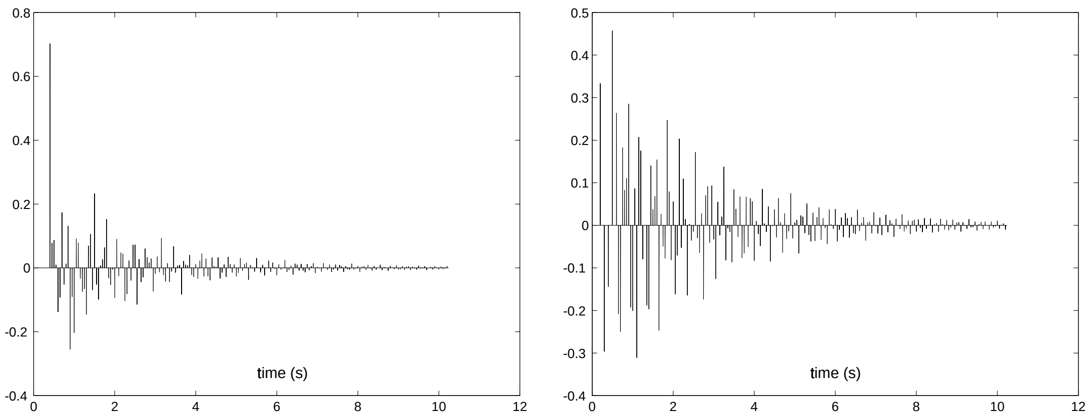
where $\Re$ denotes the real part. Equations (9) and (17) formulate relations between the reflection and transmission responses for the situation without (R_0, T_0) and with (R, T) free surface multiples, respectively.

Obtaining the reflection response, including its coda, from the transmission response

Equation (17) shows that the real part of the reflection response can be obtained from the transmission response. Since the reflection response is causal, the imaginary part can be obtained via the Hilbert transform of the real part. Alternatively, $2\Re\big[R(\mathbf{x}_A, \mathbf{x}_B, \omega)\big]$ can be transformed to the time domain and subsequently be multiplied by the Heaviside step-function, yielding the time domain version of $R(\mathbf{x}_A, \mathbf{x}_B, \omega)$. We illustrate this with a one-dimensional example, i.e., we consider plane wave responses of a horizontally layered medium. For this situation equation (17) simplifies to

$$2\Re\big[R(\omega)\big] = 1 - T^*(\omega)T(\omega). \tag{18}$$

In our example, the medium consists of 7 layers, each with a layer thickness of 100 m, a constant density of 1000 kg/m^3 and propagation velocities of 1000, 2000, 1000, 2000, 4000, 2000 and 4000 m/s. The flux-normalized transmission response including free surface multiples is shown in the time domain in Figure 2a. Employing equation (18), transforming the result to the time domain and taking the causal part yields the reflection response, as shown in Figure 2b. This result perfectly matches the directly modeled reflection response (not shown).



(a) (b)

Figure 2: *(a) Plane wave transmission response of a horizontally layered medium with free surface. (b) Reflection response, obtained from (a) by employing equation (18) and the causality principle.*

Obtaining the coda of the transmission response from the reflection response

Obtaining the transmission response from the reflection response is less trivial, because T and T_0 do not appear in isolated form in equations (9) and (17). We will make use of the fact that T_0, i.e. the transmission response without free surface multiples, can be thought to be composed of a primary transmission response and a causal coda. We will not discuss how this coda can be obtained for general configurations from the reflection response via equation (9). Instead we consider again the one-dimensional situation, for which equation (9) simplifies to

$$T_0^*(\omega)T_0(\omega) = 1 - R_0^*(\omega)R_0(\omega). \tag{19}$$

Defining $T_0(\omega) = \mathcal{P}(\omega)\mathcal{C}(\omega)$, where $\mathcal{P}(\omega)$ is the flux-normalized primary transmission response (with $\mathcal{P}^*(\omega)\mathcal{P}(\omega) = 1$) and $\mathcal{C}(\omega)$ is the coda, we obtain

$$\mathcal{C}^*(\omega)\mathcal{C}(\omega) = 1 - R_0^*(\omega)R_0(\omega). \tag{20}$$

Following the O'Doherty-Anstey approach (Geophys. Prosp., 1971), we write

$$\mathcal{C}(\omega) = \exp\{-\mathcal{A}(\omega)\Delta x_3\}, \tag{21}$$

where $\Delta x_3 = x_{3,m} - x_{3,0}$ and $\mathcal{A}(\omega)$ is the Fourier transform of the causal part of the auto-correlation of the reflectivity. Substitution in equation (20) and taking the logarithm of both sides gives

$$2\Re\big[\mathcal{A}(\omega)\big]\Delta x_3 = -\ln\big[1 - R_0^*(\omega)R_0(\omega)\big]. \tag{22}$$

Figure 3a shows the plane wave reflection response of the same medium as before, without free surface multiples. Employing equation (22), applying the causality principle and substituting the result in equation (21) yields the coda of the transmission response, as shown in Figure 3b. This result perfectly matches the directly modeled transmission coda (not shown).

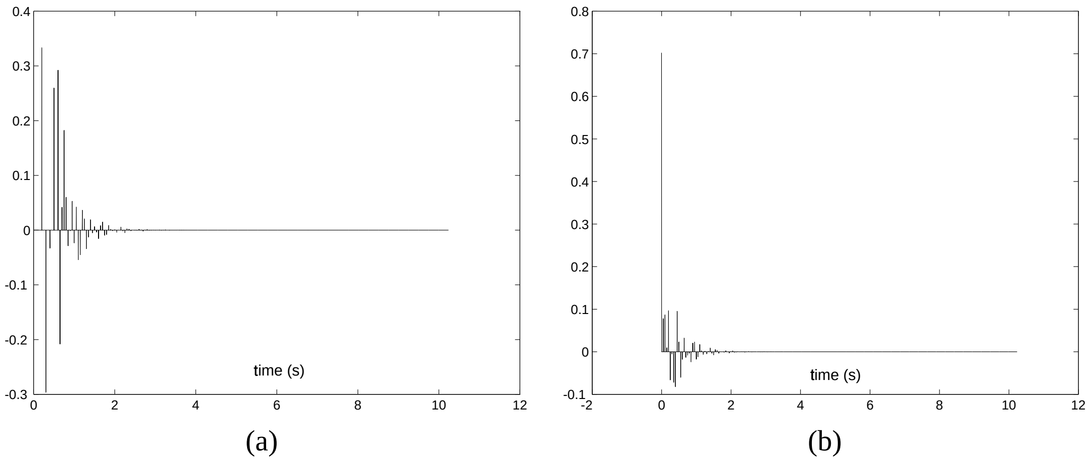


Figure 3: *(a) Plane wave reflection response of a horizontally layered medium without free surface. (b) Coda of transmission response, obtained from (a) by employing equation (22) and the causality principle.*

Discussion

Equation (18) is the basis for what is called 'acoustic daylight imaging' (Rickett and Claerbout, SEG, 1999). By correlating the response at the surface of natural noise sources in the subsurface, one obtains $T^*(\omega)T(\omega)$ (apart from a factor related to the source energy) and, by subsequently applying the causality principle, the reflection response. For the 3-D extension of this principle, it has been conjectured that "by cross-correlating noise traces recorded at two locations on the surface, we can construct the wave field that would be recorded at one of the locations if there was a source at the other" (Rickett and Claerbout, SEG, 1999). Equation (17) formulates this principle quantitatively. Bearing in mind that $T(\mathbf{x}, \mathbf{x}_A) = T(\mathbf{x}_A, \mathbf{x})$, the transmission responses in equation (17) can be seen as responses of sources in the subsurface, measured by receivers at $\mathbf{x}_A$ and $\mathbf{x}_B$ at the free surface. Equation (17) also shows that a simple correlation of the responses of a single source does not suffice, but that an integral for all source positions $\mathbf{x}$ at $\partial\mathcal{D}_m$ should be carried out. Of course this is not possible in practice, but the integral can possibly be replaced by a sum over uncorrelated sources with a random distribution, not necessarily confined to $\partial\mathcal{D}_m$. This statement is supported by the results of modeling studies of Rickett and Claerbout (SEP-92, 1996).

Equations (9) and (19) are the basis for obtaining the transmission coda from the reflection response and can be of use in seismic imaging. Imaging operators are generally formulated in terms of primary propagators in a macro model. When these propagators are supplemented with the transmission coda and subsequently inverted, one obtains improved imaging operators that account for internal multiple reflections (Wapenaar and Herrmann, SEG, 1993). Note that in this approach a macro model is still needed to account for the primary propagation effects; however, the information about the internal multiples comes from the reflection data.

In both approaches we have assumed that the medium is homogeneous below the surface $\partial\mathcal{D}_m$. Since this is not the case in practice, all results discussed above have to be applied with care. For example, time-windows may be used to suppress reflections from reflectors below $\partial\mathcal{D}_m$, but these windows will suppress also a part of the coda. A further discussion is beyond the scope of this paper.