REMOVAL OF LOVE WAVES, USING A DATA DRIVEN APPROACH

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INTRODUCTION

Due to the low velocity of shear waves, SH-wave (Shear Horizontal) reflection seismics provide a promising tool for imaging the shallow subsurface. However, SH-wave shot records often contain shot-generated noise, of which the largest contribution is due to Love waves, which are a kind of surface waves. Several characteristics of Love waves make it difficult to remove them from the data. The most important ones are that the Love wave speed is almost equal to the shear wave velocity of the upper layers, and that Love waves are dispersive, meaning that their phase velocity is frequency dependent. A full discussion on the behaviour of Love waves can be found in Aki and Richards [1].

The procedure presented in this paper, which can be used to remove Love waves from SH-wave data, is a data-driven approach, where no assumptions about the structure of the first layer is needed. The approach is similar to that of Van Borselen et al. [2] for surface multiple elimination in the marine case. Full wave theory and elastic reciprocity are used. Reciprocity is a tool which can relate two different states to each other. The final expressions give the data without surface effects, as a function of the recorded data.

THEORY

The equations given here are in the Laplace domain, and the Einstein summation convention is used. The wave field in an elastic medium is described by the elasto-dynamic equations [3]:

$$\partial_j \hat{\tau}_{i,j} - s\rho\hat{v}_i = -\hat{f}_i, \tag{1}$$

$$\frac{1}{2}\left(\partial_p \hat{v}_q + \partial_q \hat{v}_p\right) - sS_{p,q,i,j}\hat{\tau}_{i,j} = \hat{h}_{p,q}. \tag{2}$$

Here, $\hat{\tau}_{i,j}$ is the elastic stress tensor, $\hat{v}_i$ is the particle velocity, ρ is the volume density of mass, $S_{p,q,i,j}$ is the compliance tensor (the inverse of the stiffness tensor $C_{i,j,p,q}$), $\hat{f}_i$ is the volume source density of external forces, $\hat{h}_{p,q}$ is the volume source density of deformation, and finally, s is the Laplace parameter.

Two different states, called state A and state B, can be related to each other with the following scalar interaction quantity: $\partial_j(\hat{\tau}^A_{k,j}\hat{v}^B_k - \hat{\tau}^B_{k,j}\hat{v}^A_k)$. The elasto dynamic equations for state A and B are substituted, and the interaction quantity is integrated over a volume, called domain $\mathbb{V}$ with boundary $\partial\mathbb{V}$ and an outward pointing normal vector ν_j. When

Gauss' theorem is applied, and the media are assumed to be reciprocal ($S_{p,q,j,k} = S_{j,k,p,q}$, the following equation is obtained:

$$\begin{aligned}&\int_{\boldsymbol{x}\in\partial\mathbb{V}} \left(\hat{\tau}^A_{k,j}\hat{v}^B_k - \hat{\tau}^B_{k,j}\hat{v}^A_k\right)\nu_j \mathrm{d}^2\boldsymbol{x} = \\ &\quad\int_{\boldsymbol{x}\in\mathbb{V}} \left[s\left(S^B_{j,k,p,q} - S^A_{j,k,p,q}\right)\hat{\tau}^A_{j,k}\hat{\tau}^B_{p,q} - s\left(\rho^B-\rho^A\right)\hat{v}^A_k\hat{v}^B_k\right]\mathrm{d}^3\boldsymbol{x} + \\ &\quad\int_{\boldsymbol{x}\in\mathbb{V}}\left[\hat{f}^B_k\hat{v}^A_k + \hat{h}^B_{j,k}\hat{\tau}^A_{j,k} - \hat{f}^A_k\hat{v}^B_k - \hat{h}^A_{j,k}\hat{\tau}^B_{j,k}\right]\mathrm{d}^3\boldsymbol{x}.\end{aligned} \tag{3}$$

This is the global form of the Betti-Rayleigh reciprocity theorem.

From here onward, only the two-dimensional case, where SH-waves are decoupled from the other two wave types, is considered. In this case, only the crossline component $\hat{v}_2$ is of concern. Figure 1 shows a graphical representation of the two states that are used. Note that the positive x_3 direction is pointing downward. One state is the field case,

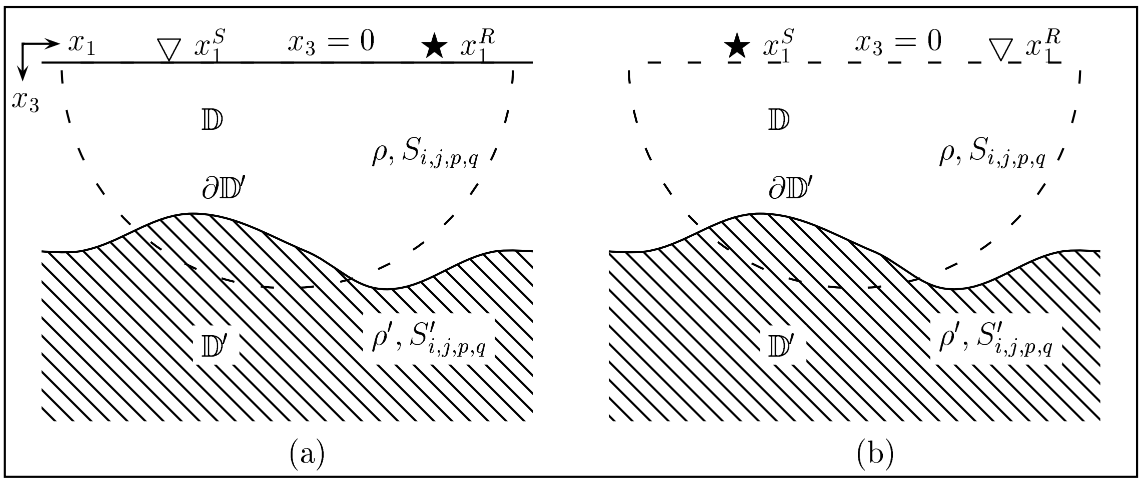

Figure 1: *The two states for the reciprocity theorem, with* $\mathbb{V} = \mathbb{D} \cup \partial\mathbb{D}' \cup \mathbb{D}'$. *a) with stress-free surface, b) without surface. The surface* $x_3 = 0$ *and the dashed half circle are the path of integration, the radius of the latter goes to infinity.*

i.e. the recorded data, with surface effects. The source is located at the surface at x_1^R. Since a *volume* source density is hard to define on the surface, the source is introduced as a boundary traction on the stress free surface, $\hat{\tau}^{\mathrm{surf}}_{2,3}(x_1, 0, s) = \hat{t}^{\mathrm{surf}}_2(s)\delta(x_1 - x_1^R)$. The other state has no surface, but a homogeneous upper half space, and obviously no surface effects are possible. The source is located at x_1^S. In this case, a volume source density can be defined normally: $\hat{f}^{\mathrm{nosurf}}_2(s)\delta(x_1 - x_1^S)\delta(x_3)$. Note that the medium parameters in $\mathbb{V}$ for both states are identical. Substituting these states in eq. (3), where state A has the superscript *surf*, and state B the superscript *nosurf*, the following equation is obtained:

$$\begin{aligned}&\int_{x_1\in\mathbb{R}} \hat{\tau}^{\mathrm{nosurf}}_{2,3}(x_1, 0|x_1^S, 0, s)\hat{v}^{\mathrm{surf}}_2(x_1, 0|x_1^R, 0, s)\mathrm{d}x_1 = \\ &\quad\frac{1}{2}\hat{f}^{\mathrm{nosurf}}_2(s)\hat{v}^{\mathrm{surf}}_2(x_1^S, 0|x_1^R, 0, s) + \hat{t}^{\mathrm{surf}}_2(s)\hat{v}^{\mathrm{nosurf}}_2(x_1^R, 0|x_1^S, 0, s).\end{aligned} \tag{4}$$

The factor $\frac{1}{2}$ is a result of integrating precisely over a delta function. The boundary integral at infinity yields zero, due to causality [4].

The term $\hat{\tau}_{2,3}^{\text{nosurf}}$ in eq. (4) has to be rewritten with the help of eq. (2). Assuming an isotropic medium: $\hat{\tau}_{2,3}^{\text{nosurf}} = (\mu/s)\partial_3 \hat{v}_2^{\text{nosurf}}$. where, μ is defined as the shear modulus. In order to apply the vertical derivative, the operations are applied in the horizontal Fourier domain. There, the differentiation becomes a multiplication with either $+\gamma_s$ or $-\gamma_s$, depending on a differentiation of an upgoing or downgoing field, respectively. γ_s is defined as $\sqrt{\frac{s^2}{c_s^2} + k_1^2}$, where c_s is the shear-wave velocity of the top layer, and k_1 the horizontal wave number.

$\hat{v}_2^{\text{nosurf}}$ can be split into an incoming and reflected wave field. The vertical derivative of the incoming wave field is zero at the surface. Therefore, it is found that: $(\mu/s)\partial_3 \hat{v}_2^{\text{nosurf}} \rightarrow \frac{\mu\gamma_s}{s}(\tilde{v}_2^{\text{nosurf}} - \tilde{v}_2^{\text{inc}})$. Note that $\tilde{v}_2^{\text{nosurf}} - \tilde{v}_2^{\text{inc}}$ is the reflected wavefield, which clearly is a purely upgoing field. $\tilde{v}_2^{\text{inc}}$ can be written as: $\tilde{v}_2^{\text{inc}} = s\hat{f}_2^{\text{nosurf}}(s)e^{-jk_1 x_1^S}/(2\mu\gamma_s)$. Next, the traction is defined as the opposite of force: $\hat{t}_2^{\text{surf}}(s) = -\hat{f}_2^{\text{nosurf}}(s) = -\hat{f}_2(s)$. Finally, after applying physical reciprocity and Parsevals theorem, eq. (4) becomes:

$$\frac{1}{2\pi}\int_{k_1\in\mathbb{R}} \frac{\mu\gamma_s}{s\hat{f}_2(s)} \tilde{v}_2^{\text{nosurf}}(k_1, 0|x_1^S, 0, s)\tilde{v}_2^{\text{surf}}(x_1^R, 0|k_1, 0, s)\mathrm{d}k_1 = \hat{v}_2^{\text{surf}}(x_1^R, 0|x_1^S, 0, s) - \hat{v}_2^{\text{nosurf}}(x_1^R, 0|x_1^S, 0, s). \quad (5)$$

This is an integral equation of the second kind. Notice that the quantities to be known are: the measured data with surface effects ($\hat{v}_2^{\text{surf}}$), the wavelet ($\hat{f}_2(s)$), and the material parameters of the top layer (via $\mu\gamma_s$). No model is needed for the structure of the first layer.

After transforming eq. (5) back to the space domain and discretization, the last equation can be written as a matrix equation, which can be solved with a direct matrix inversion:

$$\hat{\mathbf{V}}_2^{\text{surf}} = \left[\mathcal{F}^{-1}\left\{\frac{\mu\gamma_s}{s\hat{f}_2(s)}\mathcal{F}\left\{\hat{\mathbf{V}}_2^{\text{surf}}\right\}\right\}\Delta x_1 + \mathbf{I}\right] \cdot \hat{\mathbf{V}}_2^{\text{nosurf}}, \quad (6)$$

where the Fourier transformations (denoted by the symbol $\mathcal{F}$), have to be applied to the shot coordinates. Figure 2 shows the organization of these matrices. The term with the Fourier transformations has been symbolized by $\hat{\mathbf{K}}$.

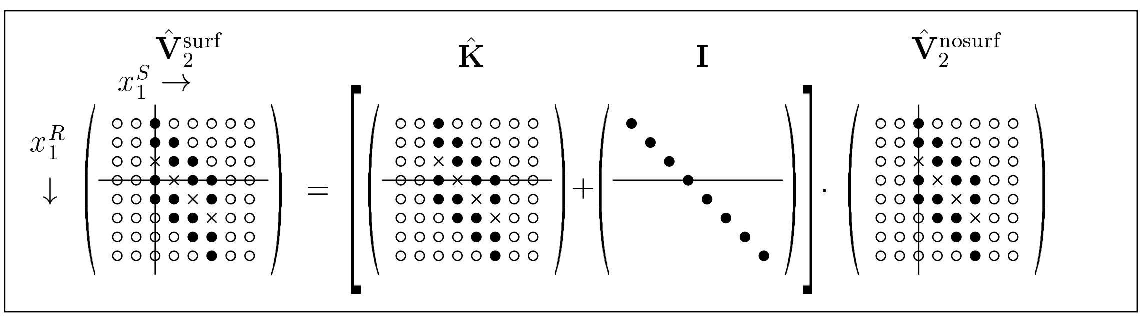

Figure 2: *Procedure for solving a set of linear equations in the Laplace domain to obtain one Love-wave-free common shot gather. A circle represents a padded zero, a disk a receiver and a cross a shot position.*

RESULTS

The results obtained so far are shown here. A dataset was made by finite difference modeling, as developed by Falk [5]. A lateral invariant model was used. First, there is

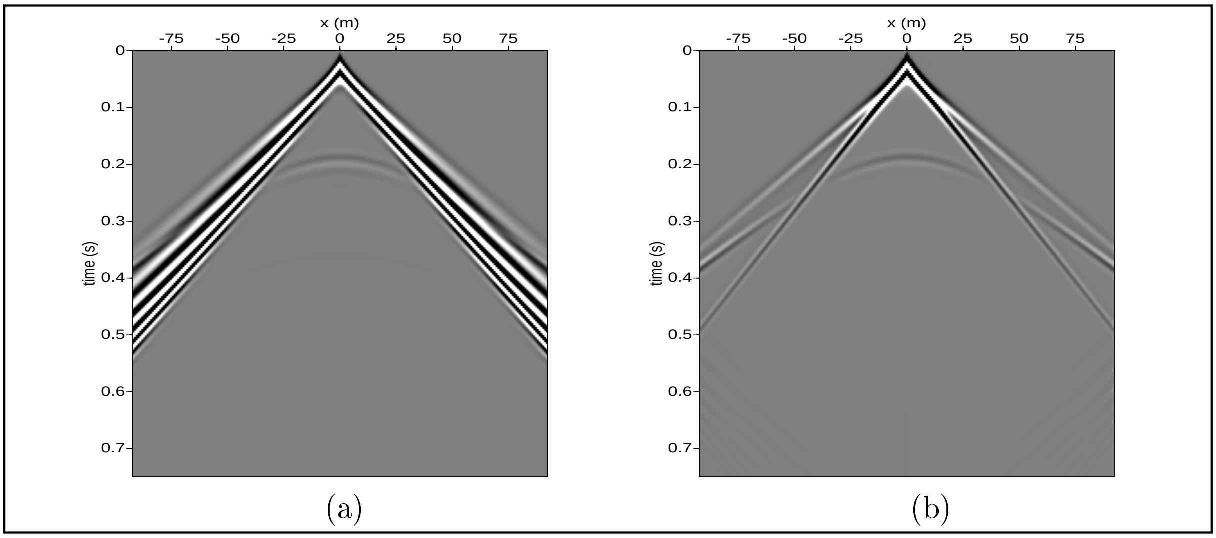

Figure 3: *Removing a Love wave, a) input data, obtained with finite difference modeling, b) result of the removal procedure.*

a layer of 1.2 m with a shear-wave velocity of 200 m/s, then a layer of 22.0 m with a shear-wave velocity of 300 m/s and finally the lower half space, which has a shear-wave velocity of 350 m/s. The source and receivers are placed on the surface. Figure 3a) shows the modeled dataset. Love waves are the most obvious event. They obscure the reflection of the deeper layer at larger offsets.

For the implementation of eq. (6), a complex Laplace parameter was used: $s = \varepsilon + j\omega$, where ω is the radial frequency, and an independent value of $\varepsilon = 6.0$ was chosen. The input data are tapered with a cosine taper, and only the non-tapered parts are shown.

Figure 3b) shows the result of the removal procedure. The amplitude is clipped differently to provide a better view. The Love waves have been removed. The reflection of the deeper layer has become more visible.

CONCLUSIONS

A technique wich removes Love waves form SH-wave data without the need of a structural subsurface model has been presented in this paper. As in the acoustic case, the source wavelet and the material parameters of the top layer are needed. The technique has been applied successfully on a simple horizontally layered medium.

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