Hierarchical decomposition and its application in seismic inversion and migration

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Summary

Hierarchical decomposition is the process of decomposing the actual wave field into a set of scattered wave fields which account for an increase of the complexity of the scattering domain with respect to the embedding. Unlike the linearization process in iterative Born-type inversion methods (like the distorted-wave Born method), hierarchical decomposition is *data-driven*, that is, the decomposition process is guided by the information that is available in the data. This paper discusses the consequences for seismic inversion and migration.

Introduction

In inverse scattering one attempts to reconstruct the material composition of a domain whose interior is inaccessible to direct measurements by probing it from the outside. To this end the domain is considered as a contrasting domain in a known background configuration. The probing is carried out by exciting the object with a number of sources, while the resulting wave field is detected at a number of receiver positions. In the corresponding mathematical description of the experiment the wave field quantities are subject to a spatial-temporal differential operator, and to the boundary conditions, such as, for example, source conditions and the radiation conditions.

In general terms inversion can be formulated as a non-linear expression where the measurements are related to the contrast function in the medium. This representation is equivalent to a volume integral over the contrasting domain where the contrast function, together with the actual field, acts as weights on the kernel function.

This kernel function depends on the position of two points in the contrasting domain and is known as the Green's function. The Green's function represents the inverse of the differential operator. In the usual formulation of the inverse problem the wave-theoretical character of the inverse operator is predetermined; only the constitutive parameters are allowed to vary. In this sense inversion is equal to inverse forward modeling. However, this approach leads to a restriction on the inversion process. The data to be inverted are harnessed due to this assumption. The parameters do not have enough flexibility to compensate for the discrepancies between the observed and calculated data when the observed data cannot be attributed to such a wave problem. In the hierarchical approach it is proven that any wave problem can be decomposed into a set of subproblems. By arranging this set of subproblems in increasing order of complexity the associated inverse process is divided into two steps. The first step consists of determining the contribution of the sub-set members to the whole data set. In the second step a linear inversion is performed on each sub-set member. In this process the influence of less complex and previously determined members is taken into account. This procedure is not equal to inverse forward modeling.

The hierarchical decomposition approach has been introduced by Fokkema (1991). In this paper we review this approach and we apply it to "two-way" as well as "one-way" wave fields.

The wave field operator formalism

To discuss the inversion problem in general terms we introduce an operator formalism that is representative for the system of first-order partial differential equations that govern the pertinent wave problem. In particular,

$$\hat{\underset{\sim}{K}}^A \mathbf{P}^A = \mathbf{S}^A,$$ (1)

where $\hat{\underset{\sim}{K}}^A$ is the actual wave field operator in the frequency domain. The quantity $\mathbf{P}^A$ represents the actual wave field vector, while $\mathbf{S}^A$ is the actual source vector.

In general the wave field operator $\hat{\underset{\sim}{K}}^A$ is complicated and generally not known. However, two constituents can be distinguished: the spatial differential operator $\hat{\underset{\sim}{D}}^A$ and the material operator $\hat{\underset{\sim}{B}}^A$:

$$\hat{\underset{\sim}{K}}^A = \hat{\underset{\sim}{D}}^A - \hat{\underset{\sim}{B}}^A.$$ (2)

To make the theory developed in the next sections and the connection with the existing inversion theories transparent we restrict ourselves in the further analysis to a fixed structure of the spatial differential operator, that is

$$\hat{\underset{\sim}{K}}^A = \hat{\underset{\sim}{D}} - \hat{\underset{\sim}{B}}^A.$$ (3)

An overview of the wave field vector, the source vector and the operators is given in Table 1, for the acoustic "two-way" as well as for the acoustic "one-way" approach.

Inverse scattering problem

In the inverse scattering problem the actual wave field $\mathbf{P}^A$ is generated by a known source distribution located in $\mathcal{D}_S$ outside the scattering domain $\mathcal{D}_{sc}$ and is measured in $\mathcal{D}_R$ also outside $\mathcal{D}_{sc}$ (see Figure 1). The domains $\mathcal{D}_S$, $\mathcal{D}_R$ and $\mathcal{D}_{sc}$ are occupying the portion of space that is commonly known as the embedding $\mathcal{D}_E$.
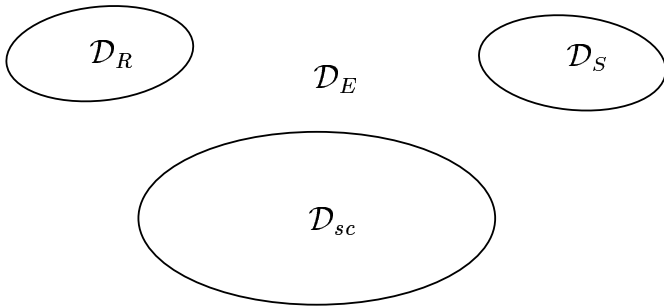

Figure 1: Scattering configuration.

The structure of the embedding is characterized by the operator $\hat{\underset{\sim}{K}}^0$. In order to formalize the inversion problem it is custom to decompose equation (1) into the following two equations

$$\hat{\underset{\sim}{K}}^0 \mathbf{P}^0 = \mathbf{S}^A$$ (4)

and

$$\hat{\underset{\sim}{K}}^0 \mathbf{P}^{A,0} = -\hat{\underset{\sim}{K}}^{A,0} \mathbf{P}^A.$$ (5)

	P	**S**	**$\hat{\underset{\sim}{D}}$**	**$\hat{\underset{\sim}{B}}$**
$\begin{pmatrix} \text{Acoustic} \\ \text{"two-way"} \end{pmatrix}$	$\begin{pmatrix} P \\ V_x \\ V_y \\ V_z \end{pmatrix}$	$\begin{pmatrix} Q \\ F_x \\ F_y \\ F_z \end{pmatrix}$	$\begin{pmatrix} 0 & \partial_x & \partial_y & \partial_z \\ \partial_x & 0 & 0 & 0 \\ \partial_y & 0 & 0 & 0 \\ \partial_z & 0 & 0 & 0 \end{pmatrix}$	$-j\omega \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & \varrho & 0 & 0 \\ 0 & 0 & \varrho & 0 \\ 0 & 0 & 0 & \varrho \end{pmatrix}$
$\begin{pmatrix} \text{Acoustic} \\ \text{"one-way"} \end{pmatrix}$	$\begin{pmatrix} P^+ \\ P^- \end{pmatrix}$	$\begin{pmatrix} S^+ \\ S^- \end{pmatrix}$	$\begin{pmatrix} \partial_z & 0 \\ 0 & \partial_z \end{pmatrix}$	$\begin{pmatrix} -j\hat{\mathcal{H}}_1 & 0 \\ 0 & j\hat{\mathcal{H}}_1 \end{pmatrix} + \begin{pmatrix} \hat{T}^+ & \hat{R}^- \\ -\hat{R}^+ & -\hat{T}^- \end{pmatrix}$ $= -j\omega\hat{\underset{\sim}{\Lambda}} \quad + \quad \hat{\underset{\sim}{\Theta}}$

Table 1: *Wave field vectors, source vectors and operator matrices for the acoustic "two-way" as well as for the acoustic "one-way" approach.*

In equation (4) $\mathbf{P}^0$ is the incident wave field that would be present in the embedding in absence of the scattering domain due to the sources in $\mathcal{D}_S$. In equation (5) $\mathbf{P}^{A,0}$ is the scattered wave field which follows from the difference between the actual and incident wave field:

$$\mathbf{P}^{A,0} = \mathbf{P}^A - \mathbf{P}^0. \tag{6}$$

When we compare equation (5) with equation (4) we conclude that the scattered wave field $\mathbf{P}^{A,0}$ can be considered as originating from the contrast source distribtion occupying $\mathcal{D}_{sc}$. This contrast source follows from the contrast operator

$$\hat{\underset{\sim}{K}}^{A,0} = \hat{\underset{\sim}{K}}^A - \hat{\underset{\sim}{K}}^0, \tag{7}$$

which operates on the actual wave field $\mathbf{P}^A$ in $\mathcal{D}_{sc}$. Using $\hat{\underset{\sim}{K}}^A = \hat{\underset{\sim}{D}} - \hat{\underset{\sim}{B}}^A$ and $\hat{\underset{\sim}{K}}^0 = \hat{\underset{\sim}{D}} - \hat{\underset{\sim}{B}}^0$ it follows that

$$\hat{\underset{\sim}{K}}^{A,0} = -\hat{\underset{\sim}{B}}^{A,0} = -(\hat{\underset{\sim}{B}}^A - \hat{\underset{\sim}{B}}^0). \tag{8}$$

From equations (4) and (5) expressions for $\mathbf{P}^0$ and $\mathbf{P}^{A,0}$ can be obtained by applying the inverse embedding operator $\{\hat{\underset{\sim}{K}}^0\}^{-1}$ to the left- and right-hand sides of the respective equations:

$$\mathbf{P}^0 = \{\hat{\underset{\sim}{K}}^0\}^{-1}\mathbf{S}^A \tag{9}$$

and

$$\mathbf{P}^{A,0} = \{\hat{\underset{\sim}{K}}^0\}^{-1}\hat{\underset{\sim}{B}}^{A,0}\mathbf{P}^A. \tag{10}$$

The inverse operator $\{\hat{\underset{\sim}{K}}^0\}^{-1}$ represents a volume integral with the Green's function of the embedding as kernel. In general this volume integral is operational throughout the entire embedding. However, due to the non-overlapping confined spatial occupation of sources and contrast, the domain of application of the volume integral reduces to $\mathcal{D}_S$ and $\mathcal{D}_{sc}$ in equations (9) and (10), respectively. It is the task of inversion to determine the contrast operator $\hat{\underset{\sim}{B}}^{A,0}$ from the measured scattered wave field $\mathbf{P}^{A,0}$ in $\mathcal{D}_R$. As such equation (10) is not suitable for this task since $\mathbf{P}^A$ also depends on the contrast, which makes the problem non-linear.

Most attempts to use equation (10) for inversion purposes are based on linearization such that the medium reconstruction is iteratively updated, with the hope that this process converges to the actual contrasting medium. First in the next section we discuss a conventional approach known as the distorted-wave Born method.

Distorted-wave Born method

In the distorted-wave Born method (see for example Beylkin and Oristaglio, 1985) the linearization is achieved by assuming the following equivalence condition

$$\hat{\underline{\mathbf{B}}}^{A,0}\mathbf{P}^A = \hat{\underline{\mathbf{B}}}^{1,0}\mathbf{P}^0,\tag{11}$$

meaning that the actual contrast operator operating on the *actual wave field* can be replaced by a first estimate of the contrast operator operating on the *incident wave field*. Then substituting equation (11) in equation (10) and using equation (9) we obtain

$$\mathbf{P}^{A,0} = \{\hat{\underline{\mathbf{K}}}^0\}^{-1}\hat{\underline{\mathbf{B}}}^{1,0}\{\hat{\underline{\mathbf{K}}}^0\}^{-1}\mathbf{S}^A.\tag{12}$$

Then, in principle, the contrast operator $\hat{\underline{\mathbf{B}}}^{1,0}$ can be determined by matching the right-hand side of the integral representation of equation (12) to the data of the scattered wave field $\mathbf{P}^{A,0}$, measured at all receiver positions in $\mathcal{D}_R$, for all source positions in $\mathcal{D}_S$ and using all frequencies. However, this is an ill-posed problem, because in equation (12) the equality sign applies at the receiver locations in $\mathcal{D}_R$, while the unknown contrast operator $\hat{\underline{\mathbf{B}}}^{1,0}$ has to be determined in $\mathcal{D}_{sc}$. The next step is to update the embedding

$$\hat{\underline{\mathbf{K}}}^1 = \hat{\underline{\mathbf{K}}}^0 + \hat{\underline{\mathbf{K}}}^{1,0} = \hat{\underline{\mathbf{K}}}^0 - \hat{\underline{\mathbf{B}}}^{1,0},\tag{13}$$

and

$$\hat{\underline{\mathbf{K}}}^1\mathbf{P}^1 = \mathbf{S}^A.\tag{14}$$

The updated scattered wave field is computed according to

$$\mathbf{P}^{A,1} = \mathbf{P}^A - \mathbf{P}^1\tag{15}$$

and the inversion formula for the next iteration is obtained as

$$\mathbf{P}^{A,1} = \{\hat{\underline{\mathbf{K}}}^1\}^{-1}\hat{\underline{\mathbf{B}}}^{2,1}\{\hat{\underline{\mathbf{K}}}^1\}^{-1}\mathbf{S}^A.\tag{16}$$

This process is repeated until after $N+1$ steps $\mathbf{P}^{A,N}$ is minimal for all receiver positions, meaning that $\hat{\underline{\mathbf{K}}}^N$ resembles $\hat{\underline{\mathbf{K}}}^A$. The updated embedding includes the scattering domain.

The equivalence relation of equation (11) has been crucial in the evolution of this iterative scheme. This assumption is also known as the first order Born approximation. However, there are no rational arguments why this condition should hold. In practice it only works when the embedding is close to the actual medium. In characterizing the method we can say that this inversion procedure

is a purely mathematical affair. At no point the data are asked for their opinion during the mathematical evolution. Consequently, the bad performance of the procedure results from the inflexibility of the mathematical framework to cope with the data. A similar discussion is true for the iterative Born method (see for example Devaney, 1982). This iterative method approximates the contrast by keeping the embedding constant but updating the wave field in the scattering domain.

To gain some insight in the complexity of the Born approximation we rewrite equation (10), using the inverse of equation (1)

$$\mathbf{P}^{A,0} = \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \hat{\underset{\sim}{\mathbf{B}}}^{A,0} \{\hat{\underset{\sim}{\mathbf{K}}}^A\}^{-1} \mathbf{S}^A. \tag{17}$$

Equation (17), like equation (10), can be also used as an integral equation for $\mathbf{P}^{A,0}$ in $\mathcal{D}_{sc}$. Then using reciprocity on the left-hand side of equation (17) between receiver and source coordinates, similar as for the surface integral representations (Fokkema et al., 1993) we obtain

$$\mathbf{P}^{A,0} = \{\hat{\underset{\sim}{\mathbf{K}}}^A\}^{-1} \hat{\underset{\sim}{\mathbf{B}}}^{A,0} \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \mathbf{S}^A, \quad \text{for} \quad \mathbf{x} \in \mathcal{D}_{sc}. \tag{18}$$

Using the result of equation (18) in the representation of equation (10) together with equation (6) we arrive at

$$\begin{aligned} \mathbf{P}^{A,0} &= \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \hat{\underset{\sim}{\mathbf{B}}}^{A,0} \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \mathbf{S}^A \\ &+ \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \underbrace{\hat{\underset{\sim}{\mathbf{B}}}^{A,0} \{\hat{\underset{\sim}{\mathbf{K}}}^A\}^{-1} \hat{\underset{\sim}{\mathbf{B}}}^{A,0}}_{\hat{\underset{\sim}{\mathbf{R}}}^{\mathcal{D}}} \{\hat{\underset{\sim}{\mathbf{K}}}^0\}^{-1} \mathbf{S}^A, \end{aligned} \tag{19}$$

which is an exact expression. The first term on the right-hand side of equation (19) represents the first-order Born term and as such the first term of the *iterative* Born method. The second term represents the correction. As can be seen, the dominant term is the intrinsic defined integral correction operator $\hat{\underset{\sim}{\mathbf{R}}}^{\mathcal{D}}$, which can be considered as a reflection operator of the scattering domain, not depending on source and receiver characteristics. This is a complicated operator involving all scattering phenomena in the actual medium. It is therefore not reasonable to hope that this operator can be expressed in a series expansion of the embedding operator, unless the embedding is close to the actual medium.

Hierarchical decomposition

In hierarchical decomposition our aim is to decompose the actual wave field into a set of scattered wave fields $\mathbf{P}^{n+1,n} = \mathbf{P}^{n+1} - \mathbf{P}^n$ which account for an increase of the complexity of the scattering domain with respect to the embedding, hence

$$\mathbf{P}^A = \mathbf{P}^0 + \sum_{n=0}^{N} \mathbf{P}^{n+1,n}, \tag{20}$$

with $\mathbf{P}^{N+1} = \mathbf{P}^A$. In the following we derive a set of representations for the incident field $\mathbf{P}^0$, the partial scattered fields $\mathbf{P}^{n+1,n}$ and the residual wave field $\mathbf{P}^{A,N}$. The implications for forward modeling and inversion/migration will be discussed below equation (30).

We start with the incident wave field definition of equation (4), while the residual wave field, $\mathbf{P}^{A,0}$, satisfies equation (5). Our first aim is to decompose equation (5), using $\hat{\underset{\sim}{\mathbf{K}}}^{A,0} = \hat{\underset{\sim}{\mathbf{K}}}^{1,0} + \hat{\underset{\sim}{\mathbf{K}}}^{A,1} = -(\hat{\underset{\sim}{\mathbf{B}}}^{1,0} + \hat{\underset{\sim}{\mathbf{B}}}^{A,1})$ and $\mathbf{P}^{A,0} = \mathbf{P}^{1,0} + \mathbf{P}^{A,1}$ in the left-hand side and right-hand side of equation (5), respectively. Then equation (5) decomposes into

$$\hat{\underset{\sim}{\mathbf{K}}}^0 \mathbf{P}^{1,0} = \hat{\underset{\sim}{\mathbf{B}}}^{1,0} \mathbf{P}^0, \tag{21}$$

while the new residual wave field $\mathbf{P}^{A,1}$ follows from

$$\hat{\underset{\sim}{\mathbf{K}}}^{0}\mathbf{P}^{A,1} = \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,1}\mathbf{P}^{0} + \hat{\underset{\sim}{\mathbf{B}}}^{A,0}\mathbf{P}^{A,0}\right). \tag{22}$$

Operating in this way, after n steps we arrive at

$$\hat{\underset{\sim}{\mathbf{K}}}^{n-1}\mathbf{P}^{A,n} = \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n}\mathbf{P}^{n-1} + \hat{\underset{\sim}{\mathbf{B}}}^{A,n-1}\mathbf{P}^{A,n-1}\right). \tag{23}$$

Now we increase the complexity of the medium to arrive at the $n+1$th step by rewriting equation (23) as

$$\begin{aligned}
\hat{\underset{\sim}{\mathbf{K}}}^{n-1}\mathbf{P}^{A,n} &= \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n+1} + \hat{\underset{\sim}{\mathbf{B}}}^{n+1,n}\right)\mathbf{P}^{n-1} \\
&+ \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n+1} + \hat{\underset{\sim}{\mathbf{B}}}^{n+1,n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\right)\mathbf{P}^{n,n-1} \\
&+ \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\right)\mathbf{P}^{A,n},
\end{aligned} \tag{24}$$

which leads to

$$\begin{aligned}
\hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{A,n} &= \left(\hat{\underset{\sim}{\mathbf{B}}}^{n+1,n}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\mathbf{P}^{n,n-1}\right) \\
&+ \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n+1}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{A,n}\mathbf{P}^{A,n}\right).
\end{aligned} \tag{25}$$

Decomposing the wave field using $\mathbf{P}^{A,n} = \mathbf{P}^{n+1,n} + \mathbf{P}^{A,n+1}$ leads to the *local* linearization

$$\hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{n+1,n} = \left(\hat{\underset{\sim}{\mathbf{B}}}^{n+1,n}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\mathbf{P}^{n,n-1}\right), \tag{26}$$

and the residual wave field

$$\hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{A,n+1} = \left(\hat{\underset{\sim}{\mathbf{B}}}^{A,n+1}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{A,n}\mathbf{P}^{A,n}\right), \tag{27}$$

which has the same structure as equation (23) and is the result after $n+1$ steps.

Using a generalization of equation (8) we can deduce from equation (26) the following identity:

$$\hat{\underset{\sim}{\mathbf{K}}}^{n+1}\mathbf{P}^{n+1} + \hat{\underset{\sim}{\mathbf{B}}}^{n+1,n}\mathbf{P}^{n+1,n} = \hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\mathbf{P}^{n,n-1}. \tag{28}$$

This implies that the left- and right-hand side of equation (28) have to be independent of n. Evaluating this identity for $n = 1$ and using equation (21) with $\hat{\underset{\sim}{\mathbf{B}}}^{1,0} = -\hat{\underset{\sim}{\mathbf{K}}}^{1,0}$, we conclude that the constant is equal to $\mathbf{S}^{A}$. In other words, we have

$$\hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{n} = \mathbf{S}^{A} - \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\mathbf{P}^{n,n-1}. \tag{29}$$

Note the difference in definition of the wave field $\mathbf{P}^{n}$ from the Born approach ($\hat{\underset{\sim}{\mathbf{K}}}^{n}\mathbf{P}^{n} = \mathbf{S}^{A}$): the source field in the nth step is composed of the actual source field and a contrast source field determined by the *local* contrast between the nth configuration and its predecessor. The incident field, the set of partial scattered fields $\{\mathbf{P}^{1,0}, \cdots, \mathbf{P}^{N,N-1}\}$ and the residual wave field $\mathbf{P}^{A,N}$ follow from

$$\begin{aligned}
\mathbf{P}^{0} &= \{\hat{\underset{\sim}{\mathbf{K}}}^{0}\}^{-1}\mathbf{S}^{A}, \\
\mathbf{P}^{1,0} &= \{\hat{\underset{\sim}{\mathbf{K}}}^{0}\}^{-1}\hat{\underset{\sim}{\mathbf{B}}}^{1,0}\mathbf{P}^{0}, \\
\mathbf{P}^{n+1,n} &= \{\hat{\underset{\sim}{\mathbf{K}}}^{n}\}^{-1}\left(\hat{\underset{\sim}{\mathbf{B}}}^{n+1,n}\mathbf{P}^{n} + \hat{\underset{\sim}{\mathbf{B}}}^{n,n-1}\mathbf{P}^{n,n-1}\right), \\
&\quad n = 1, \cdots, N-1, \\
\mathbf{P}^{A,N} &= \{\hat{\underset{\sim}{\mathbf{K}}}^{N-1}\}^{-1}\left(\hat{\underset{\sim}{\mathbf{B}}}^{A,N}\mathbf{P}^{N-1} + \hat{\underset{\sim}{\mathbf{B}}}^{A,N-1}\mathbf{P}^{A,N-1}\right).
\end{aligned} \tag{30}$$

Recall that the actual wave field $\mathbf{P}^{A}$ is related to these terms via equation (20):

$$\mathbf{P}^{A} = \mathbf{P}^{0} + \sum_{n=0}^{N}\mathbf{P}^{n+1,n}, \tag{31}$$

with $\mathbf{P}^{N+1} = \mathbf{P}^A$. In the direct scattering problem (forward modeling) the embedding operator $\hat{\mathbf{K}}^0$, the set of contrast operators $\{\hat{\mathbf{B}}^{1,0}, \cdots, \hat{\mathbf{B}}^{A,N}\}$ and the source $\mathbf{S}^A$ are given. The members of the wave field set $\{\mathbf{P}^0, \cdots, \mathbf{P}^A\}$ are successively computed, starting from the incident field $\mathbf{P}^0$. The choice of members of the set of contrast operators is not unique. In general they are ordered in terms of increasing complexity, such that the more complex members have a decreasing spatial support in model space.

In the inverse-scattering problem the situation is different. The system (30) as such does not constitute a linear inversion problem. It would only do so if the incident field $\mathbf{P}^0$, the set of scattered fields $\{\mathbf{P}^{1,0}, \cdots, \mathbf{P}^{N,N-1}\}$ and the source $\mathbf{S}^A$ were given. Hence the decomposition of the actual wave field into this set prior to inversion/migration is prerequisite.

Let us assume for the moment that a suitable data decomposition has been achieved. Then the contrast operators $\hat{\mathbf{B}}^{n+1,n}$ for $n = 0, \cdots, N$ are estimated by inverting the successive terms in the set of equations (30). When using the acoustic "two-way" approach this means that the medium contrast parameters $\kappa^{n+1,n}$ and $\varrho^{n+1,n}$ are estimated for $n = 0, \cdots, N$. We call this "hierarchical inversion". On the other hand, for the acoustic "one-way" approach the contrast operator $\hat{\mathbf{B}}^{n+1,n} = -j\omega\hat{\mathbf{\Lambda}}^{n+1,n} + \hat{\mathbf{\Theta}}^{n+1,n}$ is estimated for $n = 0, \cdots, N$. Note that $\hat{\mathbf{\Lambda}}^{n+1,n}$ represents an update of the macro model and $\hat{\mathbf{\Theta}}^{n+1,n}$ an update of the reflection (and transmission) operators. Hence, in this case we speak of "hierarchical migration".

The convergence speed of both approaches depends on how the data decomposition is done. An example of such a decomposition is the wavelet transformation. However, it should be noted that the system of equations (30) allows *any* decomposition. The challenge is to search for data decomposition methods that optimally guide the inversion/migration procedure.

Conclusions

In this paper we have shown that to tackle the inversion problem the linearization of the formulation and the presentation of the medium complexity can be treated in a wave-theoretical consistent way, which we have named hierarchical decomposition. In the presentation of the theory we have restricted ourselves to acoustic problems where the spatial differential operator remains fixed throughout the hierarchical decomposition. However, from the theory it can be easily seen that such a restriction is not necessary and also the spatial operator is allowed to vary during the decomposition. This feature makes it possible to adapt the decomposition to the available information in the data. Our formulation opens the way to include for example elastodynamic and Biot operators if the corresponding wave field information can be isolated from the data in terms of the partial scattered wave fields.

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