The effect of in-situ stress on the acoustic transmission response of sandstone rocks.

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Abstract

Ultrasonic experiments have revealed an interesting scaling behaviour in the acoustic tranmission response of several sandstone rock samples. The acoustic traces have been recorded for a great variety of ambient pressures that simulate the characteristic in-situ stresses to which reservoir rocks are submitted.

Some assumptions on the micro-structure and mineralogy of the rocks are made. From them, a mathematical model for the pressure-dependent scaling behaviour is derived and tested using numerical modelling techniques. Even simple one-dimensional layered models for wave propagation through the sandstone rocks show scaling behaviour on the transmission response similar to the effect observed in the experimental traces, suggesting our model as a good starting point to analyze the pressure-dependent behaviour of such reservoir rocks.

Introduction

A couple of years ago, ultrasonic transmission measurements in sandstone rock samples have been carried out at the Laboratory of Rock Mechanics, Department of Applied Earth Sciences, Delft University of Technology. An interesting scaling behaviour of the acoustic transmission response was observed. It appeared that not only the arrival time reduces when the ambient pressure increases, but that the width of the wavelet reduces by approximately the same relative amount. Changes in the amplitude of the seismic signals were also recorded with changing pressure. It seems possible that as a result of changing ambient pressure, some elastodynamic rock parameters will change, causing the observed differences in the arrival time, amplitude and width of the transmitted pulse. The aim of this work is to propose a possible theoretical explanation of the observed phenomena. First, a quick review of the experiments is outlined. After that, the analytical derivation of the scaling model and some numerical results are presented.

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Experimental data

Pressure machines and acoustic installation

Due to different sample shapes, two pressure machines were used to perform the experiments: the *uni-axial pressure machine* (for prismatic samples) with maximum applied pressure of 20 MPa, and the *tri-axial pressure machine* (only for cubic samples), which pressure range lies between 2 MPa and 82 MPa, in the three perpendicular axial directions. In each pressure plate, three piezo-electric broad-band transducers are mounted, one P-transducer and two mutually perpendicular polarized S-transducers. They have a central frequency of 1 MHz and they are in direct contact with the sample and pressed, by means of a spring, with a constant force on the block.

Rock samples

Several sandstone rock samples were used in the experiments. They are all rather clean sandstones containing quartz, feldspar and, to a lesser extent, some rock fragments. They represent typical reservoir rocks with a wide range of porosity values (5 % - 20 %). For a detailed mineralogical and petrophysical characterization of the rocks, we refer the reader to Dillen (2000).

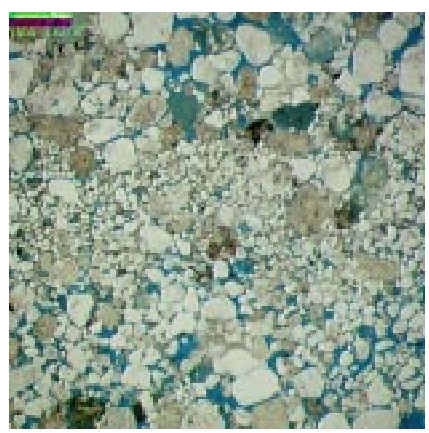

Figure 1: Thin section micro-photograph Rotliegend sandstone

In Figure 1, a micro-photograph of Rotliegend Sandstone can be seen. This thin section comes from a core drilled in a well at the northern Netherlands' off-shore, and it was not used for the ultrasonic experiments. From the matrix structure, it is possible to hypothesize that different grains can react differently to large in-situ stresses. Harder quartz grains may remain unchanged, while softer grains (i.e. feldspar, clay cement) or grain contacts may strongly change their acoustic parameters as a function of applied pressure. Also some reordering of the grains following preferable directions (for example, perpendicular the to main effective stress) can occur.

Scaling behaviour

Pressure-dependent dynamic characteristics of the sandstone rocks are fully described in Dillen (2000) and Swinnen (1997). In this work, we are specially interested in the scaling behaviour of the transmission response. It is worthwile to mention that the scaling of the traces has been observed in every sandstone sample and both in P and S waves arrivals. Also it has been carefully checked that the time and amplitude changes are not source effects, therefore it is the propagation through the sandstone that changes with changing pressure. In Figure 2, the scaling behaviour in the Rotliegend sandstone can be clearly seen. The time-axis seems to be scaled by a single factor when the ambient pressure is changed from one value to the other. Also the amplitude changes when changing the pressure.

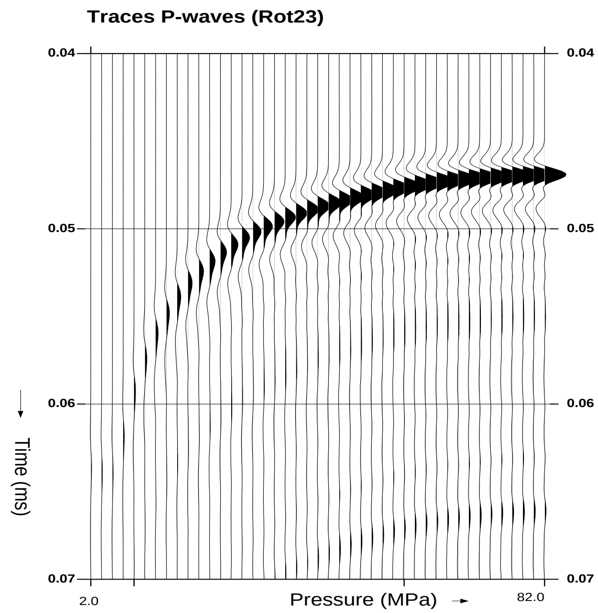

Figure 2: P-waves for increasing pressure (tri-axial pressure machine) in Rotliegend sandstone

The binary layered medium approach

In order to study the scaling behaviour analytically, we start by making a number of assumptions: we consider the sandstone horizontally layered. Secondly, the layered medium consists of only two types of material (hence the name binary layered medium), and the third assumption is that changes in the ambient pressure do not affect the layer thicknesses, but only the material acoustic parameters. Figure 3 exemplifies the assumed model for two different pressure states.

With these assumptions the depth-dependent normal-incidence plane-wave reflectivity $r(z)$ obeys the following scaling relation

$$r_B(z) = \beta\, r_A(z), \tag{1}$$

where the subscripts A and B refer to two different ambient pressure states. When the material parameters of both layer types react similarly (in a relative sense) to changes in the ambient pressure then $\beta = 1$; when they react differently, then $\beta \neq 1$. The average vertical slowness $\bar{s}$ of the material obeys the following relation

$$\bar{s}_B = \alpha\, \bar{s}_A. \tag{2}$$

In the following section we will evaluate the scaling behaviour of the transmission response of binary layered media analytically.

Scaling behaviour of the transmission response

The normal-incidence plane-wave transmission response of a layered medium can be expressed in the frequency domain in terms of a 'generalized primary' propagator $\mathcal{W}(z_1, z_0, \omega)$, according to

$$\begin{aligned} \mathcal{W}(z_1, z_0, \omega) &= \mathcal{P}(z_1, z_0, \omega)\mathcal{M}(z_1, z_0, \omega) \\ &= \exp\{-j\omega\bar{s}\Delta z\}\exp\{-\mathcal{A}(2\omega\bar{s})\Delta z\}, \end{aligned} \tag{3}$$

where $\Delta z = z_1 - z_0$. The first exponential describes the (flux-normalized) primary propagation from depth level z_0 to z_1 and the second exponential accounts for the internal multiples generated at all interfaces between those two depth levels. The function $\mathcal{A}$ is the Fourier transform of the 'causal

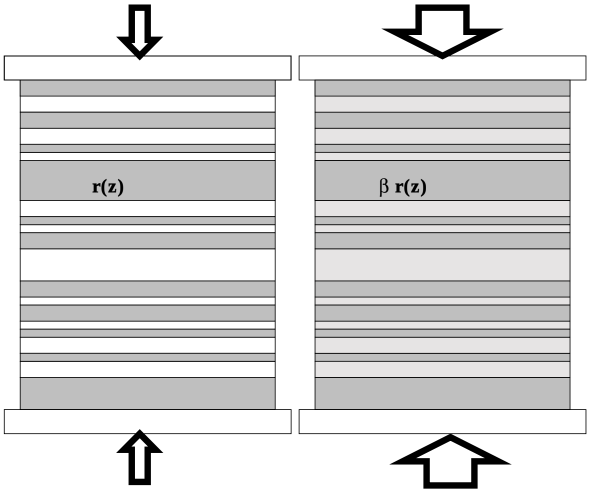

Figure 3: Two different pressure states in a binary layered medium. Left: state A (lower pressure), Right: state B (higher pressure)

part' of $\mathcal{S}(z)$, according to

$$\mathcal{A}(k) = \int_0^\infty \exp\{-jkz\}\mathcal{S}(z)\mathrm{d}z, \tag{4}$$

where $\mathcal{S}(z)$ is the autocorrelation of the reflection function $r(z)$, expressed by,

$$\mathcal{S}(z) = \frac{1}{\Delta z - z}\int_{z_0}^{z_1 - z} r(\zeta)r(\zeta + z)\mathrm{d}\zeta. \tag{5}$$

Note that equation (3) is the well-known O'Doherty-Anstey relation (O'Doherty and Anstey, 1971), except that $\mathcal{A}(k)$ in equation (4) is expressed in terms of a spatial rather than a temporal autocorrelation function. The depth-time conversion takes place in equation (3), where $\mathcal{A}(k)$ is evaluated at $k = 2\omega\bar{s}$. Assuming $r(z)$ obeys equation (1), $\mathcal{A}(k)$ has the following scaling behaviour

$$\mathcal{A}_B(k) = \beta^2\,\mathcal{A}_A(k), \tag{6}$$

where the subscripts A and B refer again to two different ambient pressure states. For these two pressure states the generalized primary propagators read

$$\begin{aligned}\mathcal{W}_A(z_1, z_0, \omega)& \\ &= \mathcal{P}_A(z_1, z_0, \omega)\mathcal{M}_A(z_1, z_0, \omega) \qquad (7)\\ &= \exp\{-j\omega\bar{s}_A\Delta z\}\exp\{-\mathcal{A}_A(2\omega\bar{s}_A)\Delta z\},\end{aligned}$$

$$\begin{aligned}\mathcal{W}_B(z_1, z_0, \omega)& \\ &= \mathcal{P}_B(z_1, z_0, \omega)\mathcal{M}_B(z_1, z_0, \omega) \qquad (8)\\ &= \exp\{-j\omega\bar{s}_B\Delta z\}\exp\{-\mathcal{A}_B(2\omega\bar{s}_B)\Delta z\},\end{aligned}$$

or, using equations (2) and (6),

$$\begin{aligned}\mathcal{W}_B(z_1, z_0, \omega)& \\ &= \exp\{-j\alpha\omega\bar{s}_A\Delta z\}\exp\{-\beta^2\mathcal{A}_A(2\alpha\omega\bar{s}_A)\Delta z\} \\ &= \mathcal{P}_A(z_1, z_0, \alpha\omega)[\mathcal{M}_A(z_1, z_0, \alpha\omega)]^{\beta^2}. \qquad (9)\end{aligned}$$

Numerical simulation

In order to test the scaling behaviour expressed by equation (9), we have performed numerical simulations for the transmission response through a binary layered medium. The total transmission response is calculated by means of forward modelling of the acoustic wave equation in the frequency domain, considering a plane-wave incident from the top and calculating the plane-wave transmitted to the bottom of the stack of horizontal parallel layers. After calculating the transmission impulse response, convolution with a Ricker wavelet is carried out.

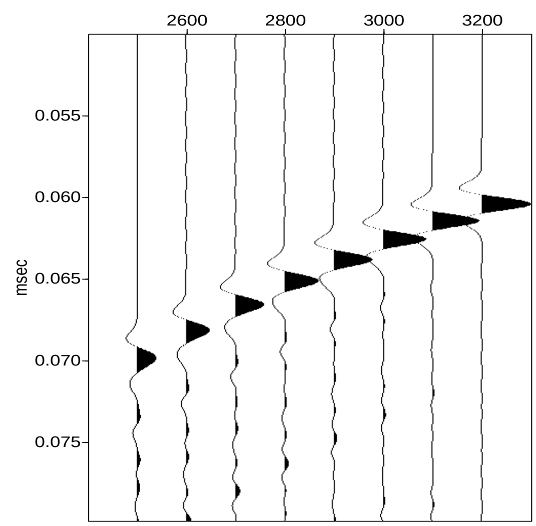

Figure 4: Transmission responses for binary layered media. Velocity of material 1 = 3500 m/s. Velocity of material 2 is labeled in the horizontal axis

In Figure 4, transmitted zero-phase Ricker wavelets through binary layered media are shown. The rightmost trace corresponds to the smallest impedance contrast and average slowness †. The other traces are calculated by fixing one velocity and decreasing the other, that is increasing the impedance contrast and the average slowness of the models. Note that the scaling behaviour in Fig-

†In all simulations, the density of the stack of layers is considered constant

ure 4 is similar to that observed in the experimental data for different ambient pressures (Figure 2). The first, third and fifth traces from the right are used to check equation (9). The direct path delay is subtracted from both responses according to the corresponding time delay for each model calculated via forward modelling. Thus equation (9) simplifies to

$$\mathcal{M}_B(z_1, z_0, \omega) = [\mathcal{M}_A(z_1, z_0, \alpha\omega)]^{\beta^2}. \qquad (10)$$

From the model velocities and using equations (1) and (2), it is possible to calculate the scaling parameters α and β to scale the fastest transmission response (3500 - 3200 m/s) to the next arrivals (3500 - 3000 m/s and 3500 - 2800 m/s) in Figure 5.

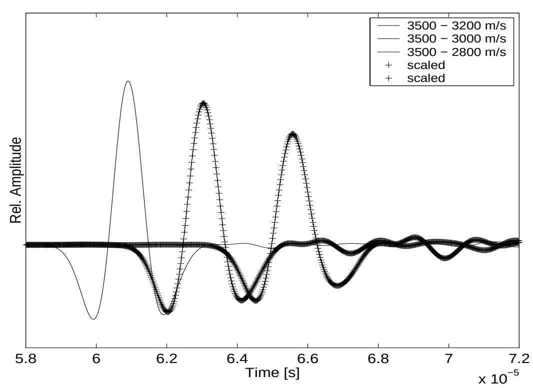

Figure 5: Detail of the first, third and fifth traces (from the right) of Figure 4 and scaled versions from the fastest arrival to the other two using equation (9).

As follows from the latter formulation, the scaling model must be applied to the transmission impulse response, that is, with no source function included. As a first step in our forward modelling algorithm, the plane-wave transmission response through the stack of layers is calculated. Then the scaling relation (equation (10)) is applied. Finally, the transmission impulse responses are convolved with a Ricker wavelet and shifted back to the correct arrival time. In Figure 5 the results are displayed. The match between the scaled versions and the calculated transmission responses is almost exact. This is another numerical confirmation of the O'Doherty-Anstey relation, although we know it is only an approximation to the total transmission response through a layered sequence.

Discussion and conclusions

The attenuation and dispersion of seismic energy due to superposition of internal multiples may well be the cause of the scaling of the transmitted wavelets through a rock sample under varying ambient pressure. The wavelet scaling produced by changes in the impedance contrast between layers corresponds to the pressure-dependent scaling behaviour observed in the experimental data. This correspondance suggests that as the ambient pressure increases, the average slowness and the impedance contrast within the rock decrease.

Equation (9) quantifies the scaling behaviour of the transmission response. The α parameter acts on both the arrival time and the width of the wavelet when the ambient pressure is changed. The exponent β^2 in the second term accounts for the amplitude change, but has an effect on the phase as well. Since this exponent is applied to a frequency-dependent term, there is not a simple scaling relation in the time-domain.

Although we have made a number of simplifying assumptions, it is worthwhile to use equation (9) as a first approximate model for observations like those in Figure 2. The α and β parameters are directly related with acoustic characteristics of the layered model (i.e. reflectivity and average slowness). Estimating them from that type of measurements for a range of different ambient pressures gives valuable information about the pressure-dependent behaviour of the reservoir rock. However, more realistic 3-D scattering models within the rock matrix may also present the scaling behaviour of the transmission response. Present research is focused on that direction.

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