REPRESENTATIONS OF SEISMIC REFLECTION DATA

PART I: STATE OF AFFAIRS

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ABSTRACT

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For the evaluation of representations of seismic reflection data, we may distinguish between boundary and volume integral methods and between two-way and one-way methods. In part I three of the four methods are reviewed; in part II the fourth method is newly introduced (the one-way volume integral representation). The pros and cons of the various representations with respect to their application in seismic inversion are briefly evaluated.

KEY WORDS: representation, one-way, two-way, inversion.

INTRODUCTION

Historically, acoustic reflection data have been represented in two different ways: by boundary integrals and by volume integrals. All modern boundary integral representations are essentially based on the work of Huygens (1629-1695), Fresnel (1788-1827), Kirchhoff (1824-1887), Rayleigh (1842-1919) and Sommerfeld (1868-1951). The foundation of the volume integral representations was laid by Born (1882-1970). Both the boundary and the volume integral representations are traditionally based on what we call the "two-way" wave equation (i.e., the common acoustic wave equation which does not explicitly distinguish between "one-way" downgoing and upgoing waves. In this paper, these boundary and volume integral representations are briefly reviewed and their importance in seismic inversion is indicated.

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For seismic reflection experiments, the incident and scattered wave fields can often be denoted as downgoing and upgoing, respectively. For this reason, the "one-way" wave equations for downgoing and upgoing waves are widely used in seismic exploration [see Claerbout (1976) for a finite difference approach and Berkhout (1982) for an integral approach]. In this paper, two approaches are discussed that lead to different representations of seismic reflection data in which the concept of one-way wave propagation plays an essential role. The first approach starts with the two-way boundary integral representation and employs the one-way wave equations to transform this into a one-way boundary integral representation (Berkhout and Wapenaar, 1989). The second approach (discussed in part II, next issue) starts with the one-way wave equations and leads step by step to a general one-way representation theorem containing boundary and volume integrals from which the former one-way representation can be derived as a special case.

GENERAL TWO-WAY REPRESENTATION THEOREM

Consider an inhomogeneous acoustic medium, characterized by the compression modulus $K(\boldsymbol{x})$ and the mass density $\varrho(\boldsymbol{x})$, where $\boldsymbol{x}$ is the Cartesian coordinate vector (x,y,z). Outside some sphere with finite radius the medium is assumed to be homogeneous and lossless. The acoustic pressure of a source at $\boldsymbol{x}_S = (x_S, y_S, z_S)$ is denoted in the frequency domain by $P(\boldsymbol{x})$ and satisfies the acoustic two-way wave equation

$$\nabla.[(1/\varrho)\nabla P] + (\omega^2/K)P = -S\delta(\boldsymbol{x} - \boldsymbol{x}_S) \ , \tag{1}$$

where $S(\boldsymbol{x})$ is the source function and ω is the angular frequency. Our aim is to find a representation for P at a detector position $\boldsymbol{x}_D = (x_D, y_D, z_D)$. To this end we introduce a Green's function $G(\boldsymbol{x},\boldsymbol{x}_D)$ as the wave field of a unit source at $\boldsymbol{x}_D$ in an inhomogeneous reference medium with compression modulus $\overline{K}(\boldsymbol{x})$ and mass density $\overline{\varrho}(\boldsymbol{x})$. Outside a sphere with finite radius, we choose $\overline{K} = K$ and $\overline{\varrho} = \varrho$. The Green's function satisfies the following two-way wave equation:

$$\nabla.[(1/\overline{\varrho})\nabla G] + (\omega^2/\overline{K})G = -\delta(\boldsymbol{x} - \boldsymbol{x}_D) \ . \tag{2}$$

Applying the theorem of Gauss to the interaction quantity $G(\varrho^{-1}\nabla P) - (\overline{\varrho}^{-1}\nabla G)P$ and employing the two-way wave equations (1) and (2) yields

$$P(\boldsymbol{x}_D) = G(\boldsymbol{x}_S,\boldsymbol{x}_D)S(\boldsymbol{x}_S) + \oint_{\partial V} [(1/\varrho)G(\partial P/\partial n) - (1/\overline{\varrho})(\partial G/\partial n)P]d^2\mathbf{x}$$

$$+ \int_V [-\omega^2(\Delta K/\overline{K}K)GP + (\Delta\varrho/\overline{\varrho}\varrho)\nabla G.\nabla P]d^3\boldsymbol{x} \ , \tag{3}$$

where $\Delta K = K - \overline{K}$, $\Delta\varrho = \varrho - \overline{\varrho}$ and where V is an arbitrary volume (containing x_S and x_D) enclosed by surface ∂V. The derivative $\partial/\partial n$ is a directional derivative in the normal direction outward from ∂V.

TWO-WAY BOUNDARY INTEGRAL REPRESENTATION

Consider the configuration shown in Fig. 1a. Here surface ∂V consists of a curved reflector Σ of infinite extent and a hemisphere Σ_0 with infinite radius in the upper half-space. We choose the reference medium continuous across Σ and equal to the actual medium throughout the upper half-space, i.e., throughout V. Now the volume integral in equation (3) vanishes. We define the wave field P as the superposition of an incident wave field P^i and a scattered wave field P^s, hence, $P = P^i + P^s$. The incident wave field is the response to the source in the absence of the reflector, i.e., in the reference medium. We then have

$$\oint_{\partial V} (1/\bar{\rho})[G(\partial P^i/\partial n) - (\partial G/\partial n)P^i]d^2x = 0 \tag{4}$$

(Sommerfeld's radiation condition, see Bleistein, 1984). Hence, applying equation (3) for the incident wave field P^i yields $P^i(x_D) = G(x_S, x_D)S(x_S)$. A result similar to equation (4) holds for P^s on the hemisphere Σ_0. Hence, applying equation (3) for the total wave field $P = P^i + P^s$, we find for the scattered wave field at x_D:

$$P^s(x_D) = \int_{\Sigma} (1/\bar{\rho})[G(\partial P^s/\partial n) - (\partial G/\partial n)P^s]d^2x \quad . \tag{5}$$

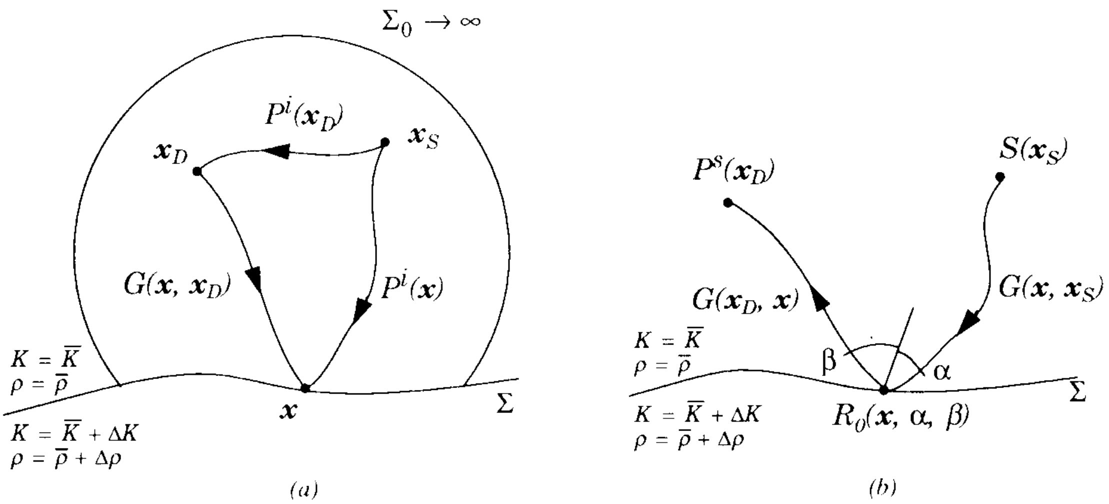

Fig. 1. a) Configuration for the two-way *boundary* integral representation; V is the upper half-space. b) The scattered field from reflector Σ is represented by a boundary integral over the reflector. The reflection coefficient $R_0(x,\alpha,\beta)$ depends on the reflector geometry as well as on the acquisition configuration.

So far no approximations have been made, however, P^s in the right-hand side has not yet been specified explicitly. In the high-frequency limit, it is common usage to relate the scattered field on Σ to the incident field on Σ by a (angle dependent) reflection coefficient $R(\boldsymbol{x}, \alpha)$, according to

$$P^s(\boldsymbol{x}) = R(\boldsymbol{x}, \alpha)P^i(\boldsymbol{x}),$$

$$[\partial P^s(\boldsymbol{x})/\partial n] = -R(\boldsymbol{x}, \alpha)[\partial P^i(\boldsymbol{x})/\partial n] \ , \boldsymbol{x} \text{ on } \Sigma \ . \tag{6}$$

This is a generalization of what is commonly known as the Kirchhoff approximation (Bleistein, 1984). Moreover, motivated by ray theory, the normal derivatives of P^i and G on Σ are generally approximated by

$$[\partial P^i(\boldsymbol{x})/\partial n] = -j\bar{k}(\boldsymbol{x})\cos\alpha(\boldsymbol{x})P^i(\boldsymbol{x}) \ ,$$

$$[\partial G(\boldsymbol{x}, \boldsymbol{x}_D)/\partial n] = -j\bar{k}(\boldsymbol{x})\cos\beta(\boldsymbol{x})G(\boldsymbol{x}, \boldsymbol{x}_D) \ , \boldsymbol{x} \text{ on } \Sigma \ , \tag{7}$$

with $\bar{k} = \omega\sqrt{(\bar{\varrho}/\bar{K})}$ and α and β being the angles of the rays with respect to the normal on Σ, see Fig. 1b. Substituting relations (6) and (7) into equation (5), using the reciprocity relation $G(\boldsymbol{x}, \boldsymbol{x}_D) = G(\boldsymbol{x}_D, \boldsymbol{x})$ and writing the incident field on Σ as $P^i(\boldsymbol{x}) = G(\boldsymbol{x}, \boldsymbol{x}_S)S(\boldsymbol{x}_S)$ yields

$$P^s(\boldsymbol{x}_D) = \int_\Sigma G(\boldsymbol{x}_D, \boldsymbol{x})R_0(\boldsymbol{x},\alpha,\beta)G(\boldsymbol{x}, \boldsymbol{x}_S)S(\boldsymbol{x}_S)d^2\boldsymbol{x} \ , \tag{8a}$$

with

$$R_0(\boldsymbol{x},\alpha,\beta) = [j\bar{k}(\boldsymbol{x})/\bar{\varrho}(\boldsymbol{x})][\cos\alpha(\boldsymbol{x}) + \cos\beta(\boldsymbol{x})]R(\boldsymbol{x},\alpha) \ , \boldsymbol{x} \text{ on } \Sigma \ . \tag{8b}$$

Note that R and R_0 depend on the reflector geometry as well as on the acquisition configuration, see Fig. 1b. Moreover, $\alpha(\boldsymbol{x})$ and $\beta(\boldsymbol{x})$ must have an unique value for each $\boldsymbol{x}$ on Σ, hence, it is implicitly assumed that the medium above Σ is relatively simple.

Interpreting the integral in equation (8a) from right to left, we encounter subsequently: propagation from the source at $\boldsymbol{x}_S$ to $\boldsymbol{x}$ on Σ, reflection at $\boldsymbol{x}$ on Σ and propagation from $\boldsymbol{x}$ to the detector at $\boldsymbol{x}_D$, see also Fig. 1b. In essence, this representation (and extensions of it) plays a central role in the seismic inversion research project at the Colorado School of Mines (Bleistein, 1984).

TWO-WAY VOLUME INTEGRAL REPRESENTATION

The starting point is again equation (3), this time for the situation that ∂V is a sphere with infinite radius, so that the surface integral vanishes. Consider the configuration of Fig. 2a. A scattering volume Ω is introduced, being the region where $\Delta K \neq 0$ and $\Delta\varrho \neq 0$. Again we define $P = P^i + P^s$ where P^i

is the incident field (i.e., the wave field in the reference medium), hence $P^i(\boldsymbol{x}_D) = G(\boldsymbol{x}_S, \boldsymbol{x}_D)S(\boldsymbol{x}_S)$, see equation (3). For the scattered wave field at $\boldsymbol{x}_D$ we thus obtain

$$P^s(\boldsymbol{x}_D) = \int_\Omega [-\omega^2(\Delta K/\overline{K}K)GP + (\Delta\varrho/\overline{\varrho}\varrho)\nabla G.\nabla P]d^3\boldsymbol{x} \ . \tag{9}$$

So far no approximations have been made, however P in the right-hand side is not yet known. Assuming the contrasts are small, it is common use to replace P by P^i. This approximation is known as the first-order Born approximation. The most important effect of this approximation is that multiple scattering is neglected. In the high-frequency approximation, the term $\nabla G.\nabla P^i$ can be approximated by

$$\nabla G(\boldsymbol{x}, \boldsymbol{x}_D).\nabla P^i(\boldsymbol{x}) = [-j\overline{k}(\boldsymbol{x})]^2 \cos\gamma(\boldsymbol{x})G(\boldsymbol{x}, \boldsymbol{x}_D)P^i(\boldsymbol{x}) \ , \tag{10}$$

with $\overline{k}^2 = \omega^2\,\overline{\varrho}/\overline{K}$ and γ being the angle between the rays at $\boldsymbol{x}$, see Fig. 2a. Substituting $P(\boldsymbol{x}) = P^i(\boldsymbol{x}) = G(\boldsymbol{x}, \boldsymbol{x}_S)S(\boldsymbol{x}_S)$ as well as equation (10) into equation (9), and using $G(\boldsymbol{x}, \boldsymbol{x}_D) = G(\boldsymbol{x}_D, \boldsymbol{x})$ yields

$$P^s(\boldsymbol{x}_D) = \int_\Omega G(\boldsymbol{x}_D, \boldsymbol{x})\Delta(\boldsymbol{x},\gamma)G(\boldsymbol{x}, \boldsymbol{x}_S)S(\boldsymbol{x}_S)d^3\boldsymbol{x} \ , \tag{11a}$$

with

$$\Delta(\boldsymbol{x},\gamma) = [-\omega^2/\overline{K}(\boldsymbol{x})][\Delta K(\boldsymbol{x})/K(\boldsymbol{x}) + \{\Delta\varrho(\boldsymbol{x})/\varrho(\boldsymbol{x})\}\cos\gamma(\boldsymbol{x})] \ . \tag{11b}$$

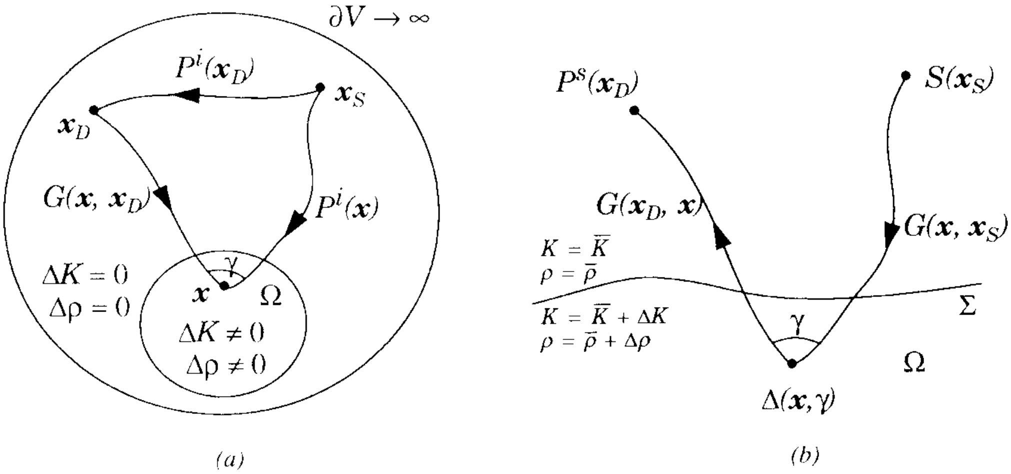

Fig. 2. a) Configuration for the two-way *volume* integral representation; V is all space.
b) The scattered field from reflector Σ is represented by a volume integral over the entire lower half-space Ω.

Note the analogy with the boundary integral representation (8a). The difference from equation (8a) becomes clear when we apply equation (11a) to the same configuration as before, see Fig. 2b. Now equation (11a) implies an integration over the entire lower half space Ω, whereas equation (8a) implies an integration along the boundary Σ only. Both representations account for (pre-critical) angle-dependent reflection effects. In essence, the volume integral representation (including higher-order terms) plays a central role in the seismic inversion research project at the Institute de Physique du Globe (Tarantola, 1984).

ONE-WAY BOUNDARY INTEGRAL REPRESENTATION

The main idea behind one-way techniques is that the "preferred direction of propagation" is along the vertical axis and that the "preferred orientation of reflectors" is horizontal. It should be noted, however, that the actual propagation direction and reflector orientation may vary from -90 to $+90$ degrees around the preferred direction/orientation. To illustrate the principle we consider the configuration shown in Fig. 3a. Here surface ∂V consists of a horizontal reflector Σ of infinite extent and a hemisphere Σ_0 with infinite radius in the upper half-space. As before, the volume integral as well as the contribution of the surface integral along Σ_0 vanishes. At the reflector Σ we define the wave field P as the superposition of a downgoing wave field P^+ and an upgoing wave field P^-, hence, $P = P^+ + P^-$. We choose the reference medium continuous across Σ and reflection free in the lower half-space, so $G = G^+$ at Σ. Finally, we assume that the medium above the detector is homogeneous and we ignore the direct wave contribution, so $P = P^-$ at x_D. Hence, from equation (3) we find

$$P^-(x_D) = \int_\Sigma (1/\bar{\varrho})[G^+\{\partial(P^+ + P^-)/\partial z\} - (\partial G^+/\partial z)(P^+ + P^-)]d^2x \quad (12)$$

Employing the one-way wave equations for $P^\pm$ and G^+ at Σ, Berkhout and Wapenaar (1989) show that the terms containing P^+ cancel (compare with equation (4)) and that the terms containing P^- are identical, hence

$$P^-(x_D) = \int_\Sigma [-2/\bar{\varrho}(x)][\partial G^-(x_D, x)/\partial z]P^-(x)d^2x \ , \quad (13)$$

where we also used the one-way reciprocity relation $G^+(x, x_D) = G^-(x_D, x)$. The upgoing wave field $P^-(x)$ on Σ can be related to the downgoing wave field $P^+(x)$ according to

$$P^-(x) = \int_\Sigma R^+(x, x')P^+(x')d^2x' \ , \quad x \text{ on } \Sigma \ , \quad (14)$$

see Berkhout (1982) or de Bruin (1993). This "pseudo-differential operator" notation is typical for the one-way approach, see also part II of this paper in the next issue. Equation (14) is a generalized convolution integral that takes the angle-dependent reflection behavior at Σ fully into account. Moreover, the reflection operator R^+ is independent of the acquisition geometry, unlike R_0,defined in equation (8b). Actually, R^+ is a true medium characterization. Substituting equation (14) into equation (13) yields

$$P^-(\boldsymbol{x}_D) = \int_\Sigma \int_\Sigma W^-(\boldsymbol{x}_D, \boldsymbol{x})R^+(\boldsymbol{x}, \boldsymbol{x}')P^+(\boldsymbol{x}')d^2\boldsymbol{x}'d^2\boldsymbol{x} \ , \tag{15a}$$

where

$$W^-(\boldsymbol{x}_D, \boldsymbol{x}) = [-2/\bar{\rho}(\boldsymbol{x})][\partial G^-(\boldsymbol{x}_D, \boldsymbol{x})/\partial z] \ . \tag{15b}$$

Note that $P^+(\boldsymbol{x}')$ may be written as $G^+(\boldsymbol{x}', \boldsymbol{x}_S)S(\boldsymbol{x}_S)$ when the source at $\boldsymbol{x}_S$ is a monopole. On the other hand, for a dipole source S_0^+ at $\boldsymbol{x}_S$ we may write

$$P^-(\boldsymbol{x}_D) = \int_\Sigma \int_\Sigma W^-(\boldsymbol{x}_D, \boldsymbol{x})R^+(\boldsymbol{x}, \boldsymbol{x}')W^+(\boldsymbol{x}', \boldsymbol{x}_S)S_0^+(\boldsymbol{x}_S)d^2\boldsymbol{x}'d^2\boldsymbol{x} \ , \tag{16a}$$

where

$$W^+(\boldsymbol{x}', \boldsymbol{x}_S) = [2/\bar{\rho}(\boldsymbol{x}_S)][\partial G^+(\boldsymbol{x}', \boldsymbol{x}_S)/\partial z_S] \ . \tag{16b}$$

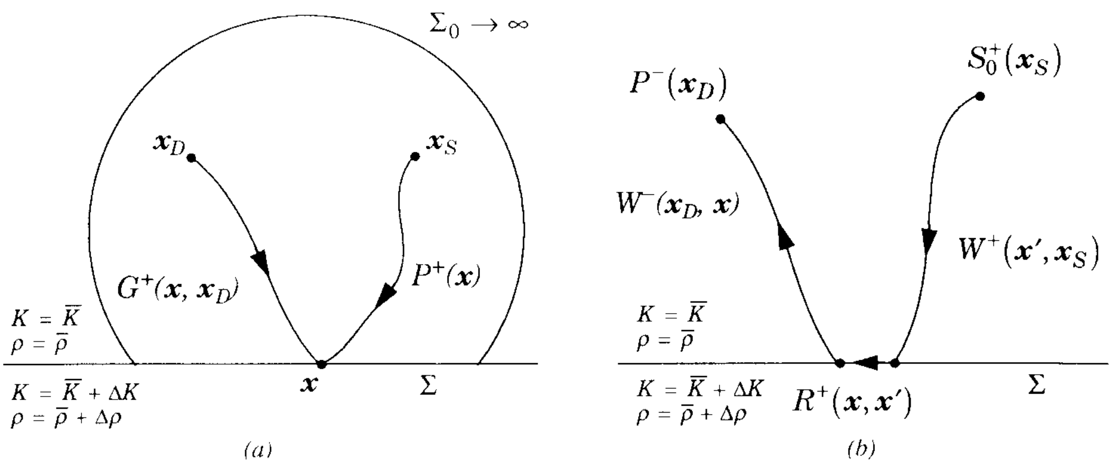

Fig. 3. a) Configuration for the one-way *boundary* integral representation; V is the upper half-space. b) The WRW model accounts for angle-dependent reflection and involves no high-frequency approximation. The reflection operator is independent of the acquisition geometry (for simplicity a horizontal reflector is shown; the WRW model is valid for 3-D inhomogeneous media, see also part II.

Representation (16a) is the integral formulation of Berkhout's so-called WRW model, see Fig. 3b. During the past twelve years, both authors have proposed several refinements, including the incorporation of multiple reflections (Berkhout, 1982; Verschuur et al., 1992) and an extension to the full elastic situation (Wapenaar and Berkhout, 1989). In essence, this representation is the basis for the research carried out in the consortium project DELPHI at Delft University.

DISCUSSION

At first sight, the two-way volume integral representation ((9) or (11a)) seems to be the most attractive starting point for inversion, since it directly relates the scattered wave field to the medium contrast parameters ΔK and $\Delta \varrho$. This representation is particularly suited for media which consist of a smooth background, with *local* inhomogeneities superimposed on it. Therefore, two-way volume integral representations find wide applications in *medical imaging* and (to a lesser extent) in *non-destructive testing* of construction materials. For seismic inversion, however,this representation is less suited, which is best illustrated by the example in Fig. 2b. Since in seismic reflection experiments the *boundaries* between the different layers are the main cause for scattering, the boundary integral representations provide a more appropriate starting point for seismic inversion. A complication of the two-way boundary integral representation (equation (8a)) is that the reflection coefficient is not an intrinsic medium property since it depends on the acquisition geometry as well. Moreover, it is implicitly assumed that the overburden is relatively simple so that the incident and scattered waves are characterized by a unique angle at each point on the reflector. The one-way boundary integral representation (equation (16a)) does not have these shortcomings and is therefore an excellent starting point for seismic inversion. The pros and cons of the various representations discussed in this paper are summarized in Table 1. It is interesting to note that, for the boundary integral methods, P^i and P^s in the two-way representation play the same role as P^+ and P^- in the one-way representation. Actually, equation (8a) can be seen as a simplified version of (16a): replacing in (16a) the reflection *operator* by a reflection *coefficient* yields (8a). Use of G or W depends on the type of source (monopole or dipole) and reflection operator (pressure to velocity or pressure to pressure). Fokkema et al. (1993) generalize (14) to curved interfaces.

In part II of this paper (next issue) a one-way volume integral representation will be derived that holds for inhomogeneous fluids and solids and that accounts for internal multiple scattering (and wave conversion). In that representation the contrast function is proportional to the vertical variations of the medium parameters so that for the situation of a single reflector the volume integral again reduces to a boundary integral.

Table 1. Pros and cons of the various representations with respect to their application in seismic inversion.

	TWO-WAY $P = P^i + P^s$	**ONE-WAY** $P = P^+ + P^-$
boundary integral methods	equation (8a) Pro: integration over a boundary Con: R depends on acquisition geometry	equation (16a) Pro: integration over a boundary Pro: R^+ independent of acquisition geometry
volume integral methods	equation (11a) Con: integration over a volume Pro: ΔK and $\Delta \rho$ independent of acquisition geometry	? (See part II)

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