MACRO MODEL ESTIMATION BY COMMON OFFSET MIGRATION AND BY SHOT RECORD MIGRATION

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ABSTRACT

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Recently, several macro velocity analysis techniques have been proposed that are based on image gather analysis. Image gathers can be obtained by common offset (CO) migration as well as by shot record (SR) migration. In this paper we point out that these two types of image gathers are significantly different. Surprisingly, several authors implicitly use CO updating equations in SR migration based velocity analysis schemes. This does not necessarily lead to erroneous macro models, but it certainly slows down the convergence speed. Hence, for an optimum performance of either CO- or SR-migration based velocity analysis schemes, it is important to employ the corresponding updating equations.

KEY WORDS: macro model, common offset migration, shot record migration, image gather, velocity estimation.

INTRODUCTION

It is well known nowadays that an accurate macro velocity model is needed for prestack depth migration. In recent years, a lot of research has been done to obtain methods that estimate macro velocity models accurately, even in situations where the assumption of hyperbolic moveout is no longer valid.

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One such method is based on *image gather analysis*. Many authors have addressed this method of velocity estimation (Al-Yahya, 1989; Cox, 1991; Etgen, 1988). Image gathers can be obtained by common offset (CO-) migration as well as by shot record (SR-) migration. In an image gather each trace represents an image of the subsurface at one and the same lateral position. If the macro velocity model is correct, all reflection events in an image gather should be horizontally aligned, irrespective of dips present in the subsurface. However, if the macro velocity model is incorrect, no horizontal alignment will occur. From the observed curvature an update for the macro velocity model can be derived.

This paper points out the importance of the choice of migration algorithm that is used to generate the image gathers. In particular, we will show that (for the same macro model) image gathers obtained by CO-migration exhibit a different misalignment curve from that obtained by SR-migration. Only when the macro velocity model is correct, are both 'curves' identical, i.e., horizontal alignment occurs in both gathers. Consequently CO-migration and SR-migration require different velocity update equations. This is ignored in the literature.

In order to obtain simple formulas, the analysis has been restricted to a horizontal reflector in a constant velocity medium, as shown in Fig. 1.

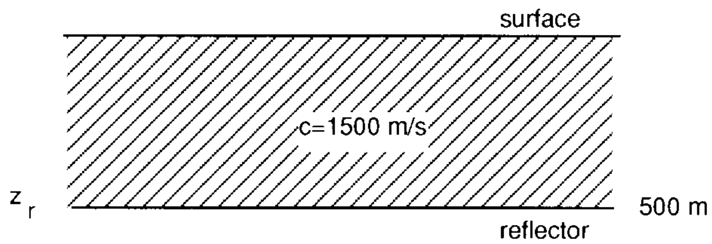

Fig. 1. Horizontal reflector in a constant velocity medium.

COMMON OFFSET MIGRATION

For one particular half offset h the observed traveltime t_{obs} in a CO-gather is constant, according to

$$t_{obs} = 2\sqrt{(h^2 + z_r^2)} / c = \text{constant}, \tag{1}$$

where z_r denotes the true reflector depth, h is half the source detector distance, and c is the true medium velocity (cf. Fig. 2).

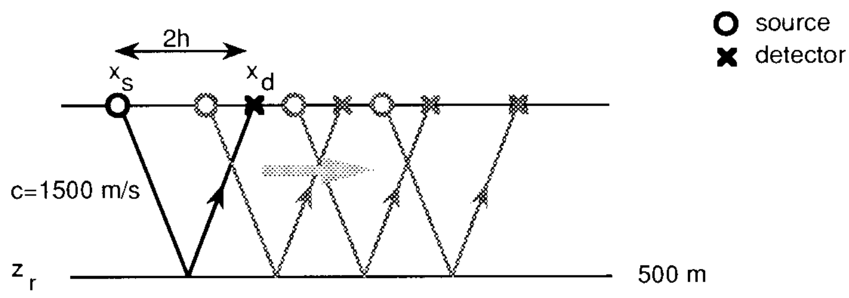

Fig. 2. Common offset geometry.

Common offset migration with velocity c_m will position the reflector at a constant depth $z_m(h)$ (Fig. 3). To find an expression for $z_m(h)$, imagine that we would use the migrated section as input for a common offset modeling experiment with velocity c_m. The modeled traveltime t_{mod} would then be equal to the observed traveltime t_{obs}. In other words, the migrated reflector would explain the observed traveltimes.

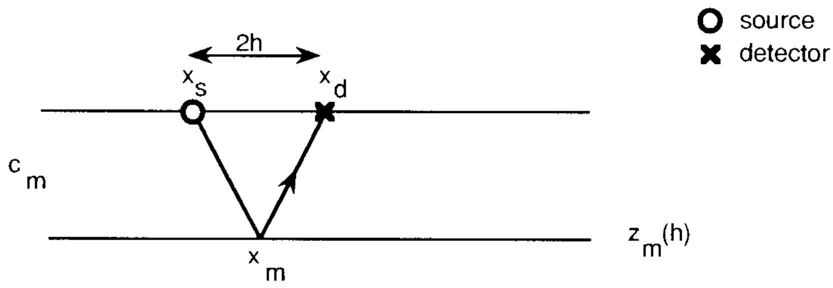

Fig. 3. Geometrical construction of the CO-migrated reflector.

Hence, common offset modeling would yield a two-way traveltime according to:

$$t_{mod} = 2 \sqrt{[h^2 + z_m^2(h)]} \;/\; c_m = \text{constant}. \tag{2}$$

Because of the above reasoning, t_{obs} in eq. (1) should be equal to t_{mod} in eq. (2), yielding

$$z_m^2(h) = \gamma^2 z_r^2 + (\gamma^2 - 1)\, h^2, \tag{3}$$

where γ represents the ratio (c_m/c) of the migration velocity to the true velocity.

Image gathers after common offset migration

Equation (3) states that the depth of the migrated reflector depends on the half offset h when the migration velocity deviates from the true medium velocity. By gathering the results for different offsets, a so-called image gather can be extracted. The curvature in the image gather after common offset migration is, for this simple subsurface, defined by equation (3), where h is now variable. Fig. 4 shows the (CO-) image gather for a migration velocity that is taken too low (c_m = 1000 m/s, c = 1500 m/s, z_r = 500 m). Note that the analytically derived curve (3), which is plotted in overlay, perfectly matches the observed curve in the image gather.

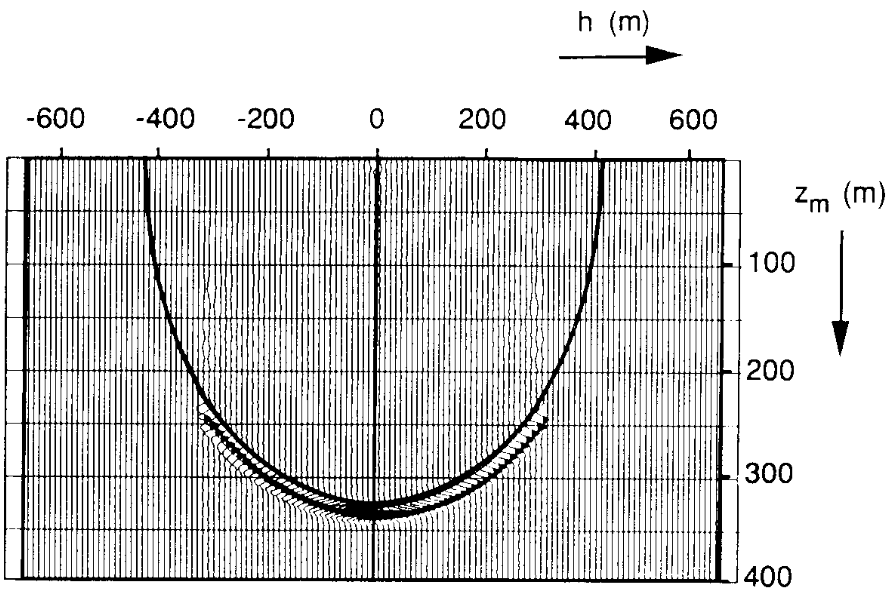

Fig. 4. Image gather obtained by Common Offset migration using a migration velocity that is too low (c_m = 1000 m/s). The curve described by equation (3) is plotted in overlay.

SHOT RECORD MIGRATION

It should be emphasized that equation (3) is derived for common offset migration. In this section we discuss the curvature that can be expected when shot record migration is used to produce the image gather.

For one particular shot record the observed traveltime t_{obs} represents a hyperbola, according to (cf. Fig. 5):

$$t_{obs} = \sqrt{[(x_d - x_s)^2 + 4z_r^2]} \;/\; c \;, \tag{4}$$

where z_r denotes the true reflector depth, $x_d - x_s$ is the source-detector distance, and c is the true medium velocity.

SR-migration with velocity c_m will give a *curved* reflector image that would again explain the observed traveltimes.

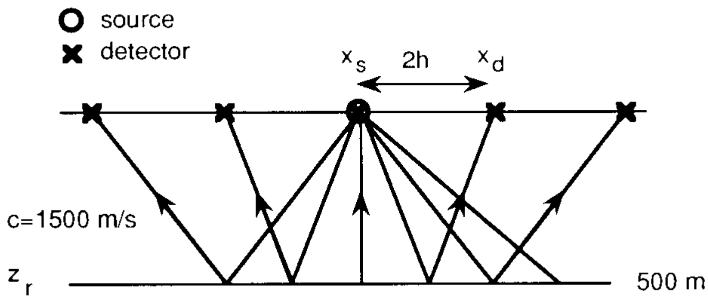

Fig. 5. Shot record geometry.

In a recent article, Maeland* (1991) derived the curvature of a migrated reflector in a parametric form according to (cf. Fig. 6):

$$x_m = x_d + z_m (x_s' - x_d) / z_s' \tag{5a}$$

$$z_m = (1/2)\, z_s' (x_s'^2 + z_s'^2 - 2x_d x_s') / (x_s'^2 + z_s'^2 - x_d x_s') \tag{5b}$$

where (x_m, z_m) denote the coordinates of a depth point on the migrated reflector, and (x_s', z_s') represent the coordinates of the virtual source, which are defined by

$$x_s' = 2h(1 - \gamma^2) \tag{6a}$$

$$z_s' = 2\gamma \sqrt{[h^2(1 - \gamma^2) + z_r^2]} = \sqrt{[c_m^2 t_{obs}^2 - 4\gamma^2 h^2]}, \tag{6b}$$

γ again being the ratio (c_m/c) of the migration velocity and the true velocity. Note that, when the migration velocity is incorrect, the coordinates of the virtual source depend on the source-detector offset at the surface. It can be verified that the traveltimes along the path from the source via the migrated reflector to the detector indeed equal t_{obs}, as defined by equation (4).

* For notational convenience, Maelands formulas are written in our notation. Note also that Maeland assumes that the source is located at $x_s = 0$.

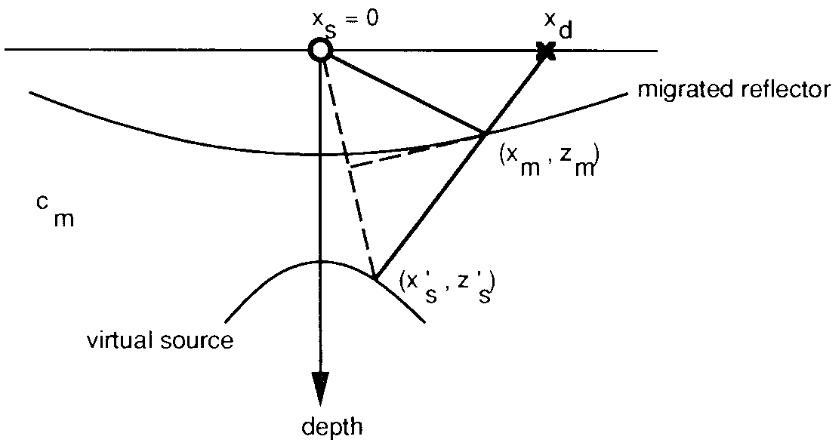

Fig. 6. Geometrical construction of the SR migrated reflector (after Maeland, 1991).

Substitution of the coordinates of the secondary source (equations (6)) into equations (5) yields (assuming this time that the source is located at $x_s \neq 0$):

$$x_m(h) - x_s = h[1 + \varepsilon(1 + h^2/z_r^2)] \tag{7a}$$

$$z_m(h) = \gamma z_r\, [1 - \varepsilon h^2/\gamma^2 z_r^2]\, \surd(1 + \varepsilon h^2/z_r^2) \tag{7b}$$

where $\varepsilon = 1 - \gamma^2$. Note that when the migration velocity equals the true medium velocity, equations (7) simplify to

$$x_m - x_s = h \tag{8a}$$

$$z_m = z_r, \tag{8b}$$

which is what we expect.

Image gathers after shot record migration

For a 1-D subsurface, image gathers (x_m is a constant; x_s is a variable) are equivalent to migrated shot records (x_m is a variable; x_s is a constant). Therefore, for the simple situation that we investigate here, equations (7) describe the curve in an image gather obtained by shot record migration. Note that when the migration velocity equals the true medium velocity, the image gather is horizontally aligned at depth z_r, which is what we expect. Fig. 7 shows the (CS-) image gather for a migration velocity that is too low (c_m = 1000 m/s). Note that the analytically derived curve (7), which is plotted in overlay, perfectly matches the observed curve in the image gather.

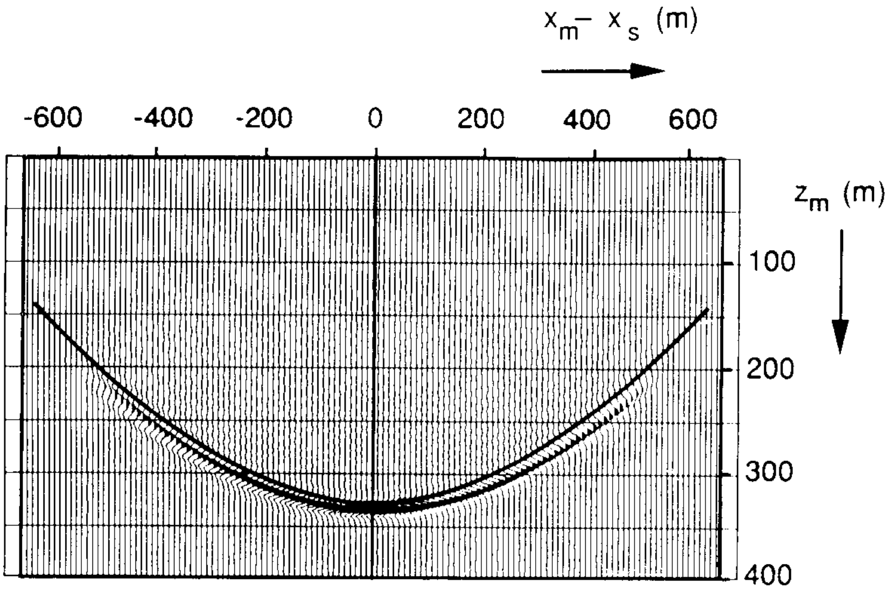

Fig. 7. Image gather obtained by Shot Record migration using a migration velocity that is too low (c_m = 1000 m/s). The curve described by equation (7) is plotted in overlay.

DISCUSSION AND CONCLUSION

In Fig. 8 the analytically derived curves for the Common Offset image gather ($h = x_m - x_s$, see Fig. 3) and the Shot Record image gather as given by equations (3) and (7) are plotted in overlay.

As can be seen, these curves deviate significantly. This implies that macro velocity estimation schemes based on SR-migration require different velocity-updating equations from such schemes based on CO-migration. This is not only true for the single reflector case as analyzed above, but for any subsurface geometry. Surprisingly, many authors employ updating equations derived for the common offset situation in shot-record-driven macro velocity estimation schemes.

To understand what actually goes wrong in those schemes, let us consider again the single reflector situation. Suppose that the SR migrated reflector in

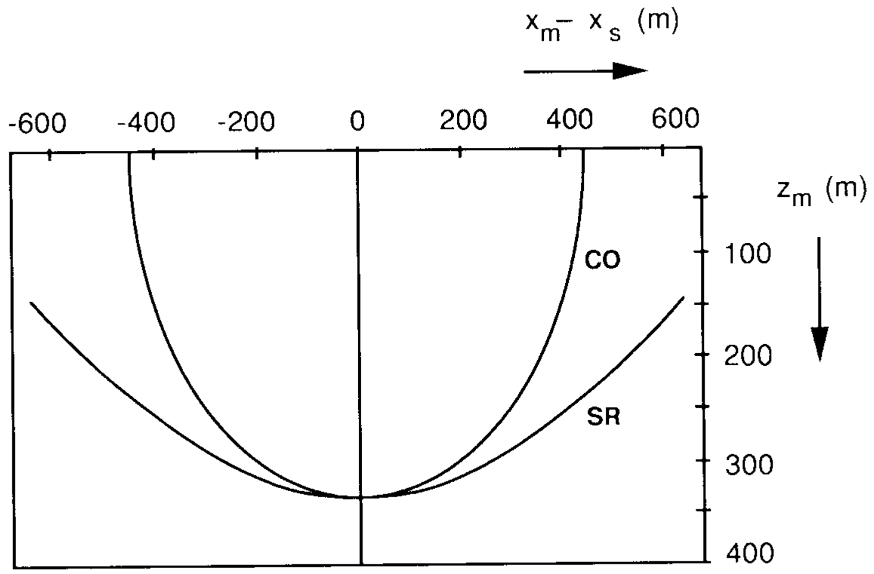

Fig. 8. Overlay of the analytic curves obtained from equations (3) and (7), for γ = 2/3.

Fig. 6 would be approximated by horizontal segments, as shown in Fig. 9. Then $(x_m - x_s)$ would equal h, just like in Fig. 3. Hence, the image gather would be described by equation (3), which was originally derived for the common offset situation. In other words, any shot-record-driven macro velocity estimation scheme that uses updating equations derived for common offset migration, implicitly violates the reflection theory (the reflection angle is no longer determined by the reflector dip, see Fig. 9). This does not necessarily lead to erroneous macro models, but it certainly slows down the convergence speed.

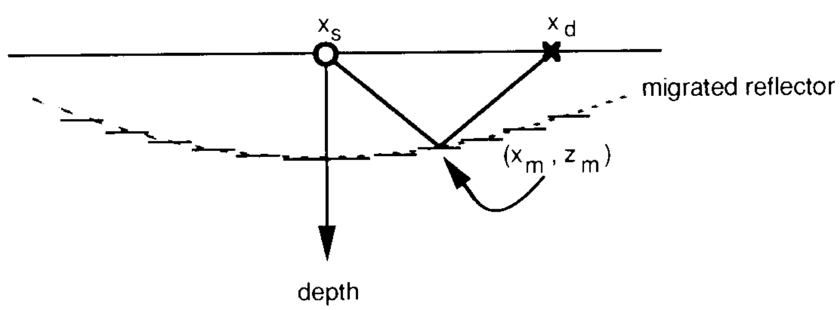

Fig. 9. Approximation of the migrated reflector by horizontal segments in the derivation of a shot record image gather. This crude approximation implicitly underlies several migration based velocity analysis schemes presented in the literature.

In conclusion, when image gather analysis is used for updating the macro velocity model, one should be aware of the differences between image gathers generated by common offset migration and those generated by shot-record migration.

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