Reply to: "Comments on the influence of wind and temperature gradients on sound propagation calculated with the two-way wave equation" [J. Acoust. Soc. Am. 91, 498–500 (1992)]

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As shown by Pierce [J. Acoust. Soc. Am. **87**, 2292–2299 (1990)], the influence of the static pressure on sound propagation through the atmosphere is only of the second order. For very accurate predictions, the improvements suggested by Raspet *et al.* [J. Acoust. Soc. Am **91**, 498–500 (1992)] can be used, but then other second-order effects and cross products should be taken into account as well. The two-dimensional Fourier transform can be used for three-dimensional sound-pressure predictions considering the method of stationary phase. Errors are well within 2% for common temperature and wind gradients, provided that the wind vector is parallel to the vertical plane through the source and receiver. For situations with a cross-wind vector full three-dimensional calculations are recommended. These situations were avoided in the original article [Nijs and Wapenaar, J. Acoust. Soc. Am. **87**, 1987–1998 (1990)].

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INTRODUCTION

The comments of Raspet *et al.*[1] on the method as we introduced it[2] can be summarized as follows: (1) The method does not take into account ambient pressure and density variations with height, and (2) The use of a one-dimensional Fourier transform is not justified. We will deal with these subjects in different sections, but we will start with the second topic.

I. THREE-DIMENSIONAL PROPAGATION PREDICTED WITH A TWO-DIMENSIONAL MODEL

In our article, we introduce a calculation method in three dimensions based on the Fourier transform from $x \Rightarrow k_x$, $y \Rightarrow k_y$, and $t \Rightarrow \omega$, defined as [see Eq. (7) of Ref. 2]:

$$P(k_x,k_y,z,\omega) = \int\int\int_{-\infty}^{\infty} p(x,y,z,t) e^{-j\omega t} e^{jk_x} e^{jk_y} dt\, dx\, dy. \quad (1)$$

This method can handle both wind and temperature effects (separately or combined) as long as the medium is built up in horizontal layers (a so-called stratified medium). The Hankel transform that is used in previous calculation methods has one disadvantage when compared to our model: It depends on only one variable r and hence requires cylindrical symmetry around the z axis. So the temperature effect, which has cylindrical symmetry, can be dealt with, but the wind effect cannot be handled separately.

After introducing the theoretical scheme for the three-dimensional method, we restricted ourselves to two-dimensional calculation schemes by leaving out the transform from $y \Rightarrow k_y$. The only reason was to save computer time and memory. However, the outcome of the two-dimensional computer calculations is used for three-dimensional sound propagation as well. We fully agree with Raspet *et al.* that there is little justification for this step in our article[2] and therefore we want to be more specific on this subject.

The justification is explained with the method of stationary phase.[3] It starts with the following relation between the sound pressure in a two-dimensional sound field (p_{2D}) and the sound pressure (p_{3D}) in a three-dimensional field:

$$p_{2D}(x,z,\omega) = \int_{-\infty}^{\infty} p_{3D}(x,y,z,\omega)\, dy. \quad (2)$$

Now p_{2D} is approximated in terms of p_{3D} by

$$p_{2D}(x,z,\omega) = C p_{3D}(x,y=y_0,z,\omega), \quad (3a)$$

with

$$C = \sqrt{2\pi j/\phi''(x,y=y_0,z,\omega)}. \quad (3b)$$

Here, $\phi(x,y,z,\omega)$ represents the phase of the complex function $p_{3D}(x,y,z,\omega)$. The primes denote differentiation with respect to y. The equation is valid in the far field. There is one important condition for this method: It can only be applied when $\phi'(x,y=y_0,z,\omega)=0$ in $y=y_0$ (hence the name of the method).

If the symmetry plane is taken in $y=0$, the method of stationary phase can be used in our own method. Temperature effects are treated correct, as their circular symmetry automatically leads to a mirror symmetry around $y=0$. Situations with wind gradients are only symmetric around $y=0$ when cross wind components are neglected. That is why we explicitly confined ourselves in the article[2] to those situations where the wind vector points in the x direction.

Now the main question of this section (can a two-dimensional method be used to solve three-dimensional problems) has reduced to the question whether or not C can be solved.

In the homogeneous free-field case solving C is easy. In that case the sound field from a monopole source situated at

$(0,0,z_s)$ is given in the Fourier domain as

$$P(k_x,k_y,z,\omega) = \exp(-jk_z|z-z_s|)/jk_z, \quad (4)$$

leaving out an arbitrary amplitude factor.

Here, k_z is given by

$$k_z^2 = k_0^2 - k_x^2 - k_y^2, \quad (5a)$$

$$k_z^2 = k_0^2 - k_x^2, \quad (5b)$$

in three and two dimensions, respectively.

The Fourier transform from k_x to x is solved numerically in our article, but for a homogeneous free field it can be solved in closed form as well (see, for instance, Ref. 4). Then, we find

$$p_{3D}(x,y,z,\omega) = (2\pi r)^{-1}e^{-jkr}, \quad (6a)$$

$$p_{2D}(x,z,\omega) = (2\pi kr)^{-1/2}e^{-j(kr+\pi/4)}. \quad (6b)$$

Actually, the 2-D solution contains a Hankel function, but this function reduces to the exponential function for the far field. The values of r, given by

$$r = [(z-z_s)^2 + x^2 + y^2]^{1/2}, \quad (7a)$$

$$r = [(z-z_s)^2 + x^2]^{1/2}, \quad (7b)$$

give the distances between source and receiver.

In a homogeneous free field case, the phase ϕ is proportional to $-kr$, and the solution of C from Eq. (3b) is found as

$$C_0 = (-2\pi jr/k)^{1/2}, \quad (8)$$

As expected C_0 gives indeed the ratio between the two solutions of Eqs. (6).

When a ground surface is added (but the medium remains homogeneous), phase shifts are introduced due to the mirror source and the value of C_0 will be slightly different from the free-field value. However, if $zz_s/r^2 \ll 1$ and $kr > 1$, these differences can be neglected. These conditions are always fulfilled in the situations described in our article.[2]

It is interesting to note that in this particular case it can be proved numerically that the two-dimensional calculation method may well be used to calculate three-dimensional sound pressures, as the computer results can be compared with previous models on sound propagation over absorbing ground surfaces. These models are basically three dimensional. For the comparison, we used the calculation method given by Attenborough *et al.*[5] The agreement was excellent; we never found differences between both methods larger then 1 dB, while these differences might be caused by computational errors as well.

The next step is to compare a case with a vertical gradient plus an absorbing ground surface with the homogeneous free-field case. Then, we find for the homogeneous case *without* a ground surface:

$$p_{2D,0}(x,z,\omega) = C_0 p_{3D,0}(x,0,z,\omega), \quad (9)$$

and when both a gradient and a ground surface are present:

$$p_{2D,g}(x,z,\omega) = C_g p_{3D,g}(x,0,z,\omega). \quad (10)$$

Dividing Eq. (10) by Eq. (9), we find

$$\frac{p_{3D,g}}{p_{3D,0}} = \frac{C_0 p_{2D,g}}{C_g p_{2D,0}}. \quad (11)$$

The value of $p_{2D,g}$ was numerically calculated in our article.[2] Here, $p_{2D,0}$ and $p_{3D,0}$ are known from the homogeneous cases, so $p_{3D,g}$ can be calculated if the quotient of C_0 and C_g is known.

In our original article,[2] we did not calculate $p_{3D,g}$, but we restricted ourselves to the so-called insertion loss (IL), defined as:

$$\text{IL} = 20 \log|p_g/p_0|. \quad (12)$$

We claimed that the insertion loss in the two-dimensional case would be approximately the same as in the three-dimensional case. From Eqs. (11) and (12), it can be concluded that this statement is true when $C_0 \approx C_g$. If, again, we assume that phase shifts from the boundary surface may be neglected, the only phase shifts causing changes in C_g come from the difference in medium parameters. For temperature gradients (with their circular symmetry), the quotient C_0/C_g will be approximately equal to $(k_g/k_0)^{1/2}$. That means that the error will be no more than 2% in extreme cases, and thus may be neglected for the cases we dealt with in the article. For wind gradients, the error can be calculated as $w/2c$, with w the wind speed and c the sound speed. This error is also 2% in extreme cases. It should be emphasized again that this calculation can be carried out only when the cross wind component is absent. When the wind vector has a component in the y direction, the phase is no longer symmetric and full three-dimensional computations are recommended.

We restricted ourselves in our article explicitly to those cases where the wind in the y direction equals zero. In outdoor measurements, however, the wind speed is mainly given as the vector in the direction from source to receiver. This must be done because it otherwise takes too long to wait for those meteorological cases where the cross wind speed equals zero. The question arises whether our model could also be expanded to cases with a non-zero cross wind speed.

De Jong did a lot of work in our laboratory examining the Parkin and Scholes measurements,[6,7] including extra measurements published in separate reports. Unfortunately, he did not publish the results in his thesis.[8] He stated that, generally speaking, the two-dimensional case can be used for those cases where the cross wind speed is no more than 1/2 the component in the direction of sound propagation. For higher ratios, the influence of the cross wind speed is too important. The vector wind speed used by Parkin and Scholes is therefore not a good indicator. This can also be concluded from the very large variation in the measured results of Parkin and Scholes when the vector wind speed in the x direction equals zero.

Our conclusion agrees with that of Raspet *et al.* that three-dimensional computations are strongly needed to calculate the influence of cross wind situations. We ourselves have no plans in that direction in the near future, but we look forward eagerly to calculation results on this subject. When there is no cross wind the method of stationary phase justifies the use of a two-dimensional method to predict three-dimensional sound propagation. The effect of temperature gradients in three dimensions can be calculated also with a two-dimensional model.

II. THE INFLUENCE OF STATIC PRESSURE IN THE ATMOSPHERE

Raspet *et al.* state in their comments[1] that density variations are not taken into account in our model. This is a misinterpretation. The sound speed is calculated [Eq. (13a) in Ref. 2] with

$$c^2 = \gamma P_0/\rho_0, \tag{13}$$

with P_0 the ambient pressure and ρ_0 the density. It is true that the pressure is kept constant along z. However, density varies with z, as it is the only possibility to introduce temperature variations into the model.

When we started our investigations described in the original article,[2] we based our theory mainly on the work of Pridmore-Brown.[9] From his theory, we decided that the static pressure is only a second-order effect for the case we wanted to investigate: near horizontal sound propagation in air. There was one other important reason not to include static pressure: We do not know of any results from measurements where a pressure gradient could be isolated, so it is impossible to compare the model with measurements.

However, although it was initially not our intention to account for ambient pressure variations, we will show that our equations are more general than might be concluded from our paper, and can be applied beyond the limits we gave. In other words, Eq. (13) can be used to account for pressure variations as well and is not restricted to near horizontal propagation.

In concept with Raspet *et al.*, we refer to the excellent article of Pierce,[10] particular to his Eqs. (23), (21), and (3), which read in our notation:

$$(1/\rho_0)\nabla\cdot(\rho_0\nabla\phi) - D_t[(1/c^2)D_t\phi] = 0, \tag{14}$$

with

$$p = -\rho_0 D_t\phi, \tag{15}$$

$$D_t = \frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla. \tag{16}$$

We quote Pierce: "Several pleasant surprises emerge during the derivation... the influence of gravity disappears, as does the influence of the ambient pressure gradients." Let us now assume a horizontal wind velocity and vertically varying ambient conditions. Then, applying a Fourier transform [as given in Eq. (1)], Eqs. (14), (15), and (16) transform to

$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0\frac{\partial\Phi}{\partial z}\right) + \left(\mu^2\frac{\omega^2}{c^2} - k_x^2 - k_y^2\right)\Phi = 0, \tag{17}$$

$$P = -j\omega\mu\rho_0\Phi, \tag{18}$$

$$D_t = j\omega\mu = j\omega(1 - w_x k_x/\omega - w_y k_y/\omega). \tag{19}$$

After substitution of Eq. (18) into (17), we obtain

$$\mu\frac{\partial}{\partial z}\left(\rho_0\frac{\partial}{\partial z}\frac{P}{\mu\rho_0}\right) + \left(\mu^2\frac{\omega^2}{c^2} - k_x^2 - k_y^2\right)P = 0. \tag{20}$$

This result is identical to Eq. (12) in our original article under the assumption that second-order derivatives with respect to z and products of first-order derivatives of the ambient parameters can be neglected. Hence, our matrix Eq. (17) which was based on Eq. (12) (both in the original article[2]), is correct to the first order also in the presence of vertical ambient conditions. According to Pierce, the approximations are of the same order as those in the well-known wave equation for an inhomogeneous medium without wind:

$$\rho_0\nabla\cdot\left(\frac{1}{\rho_0}\nabla p\right) - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0. \tag{21}$$

Equation (20) [and thus Eq. (17) of Ref. 2] is valid throughout the inhomogeneous medium. It can be solved, however, for simple gradients only. To solve it numerically, we developed a recursive extrapolation model through a medium divided into horizontal layers. To keep the propagation matrices simple, we assume each layer homogeneous, although there are also solutions with layers that vary linearly. Although ambient pressure was left out as a variable in our original article,[2] the extrapolation model is well capable to account for ambient pressure variations from layer to layer. The ambient pressure can be introduced when using Eq. (13) (given in more detail in Eqs. (13a) to (13d) of Ref. 2). Pressure variations can be taken into account just like temperature variations and our model is not restricted to near horizontal sound propagation.

The improvements on our A matrix [Eq. (19) of Ref. 2], as suggested by Raspet *et al.* are on the order $g/\omega c$ (as they state in their comments). When changing from the A matrix to the W matrix [from Eq. (19) to Eq. (33) in Ref. 2], improvements of the same order are found in the elements of the W matrix. However, these are improvements in P. The improvements in the pressure *gradient* can be estimated when a new definition of k_z is introduced. When the wind effect is left out we find in two dimensions:

$$k_z^2 = \omega^2/c^2 - g^2/c^4 - k_x^2. \tag{22}$$

In our original definition,[2] the second term on the right side was not found. The static pressure is introduced by substitution of Eq. (13) into Eq. (22). The influence of the second term on the pressure gradient is on the order $2g^2/\omega^2c^2$ when compared to the first term and thus extremely small in common sound propagation. As the value of the gradient is more important than the value of the static pressure itself for the calculation of the sound pressure, the changes suggested by Raspet *et al.* can be considered of the second order.

If, for some reason, second-order improvements are necessary, we think *all* ambient parameters should be taken into account. Second-order derivatives and cross products, neglected by Pierce throughout the derivation of Eq. (21) and by us for the derivation of Eq. (12) of Ref. 2 may well be on the order $g/\omega c$ too. Also the calculation scheme itself introduces errors of this order.

Surprisingly, Raspet *et al.* ignored temperature and wind speed gradients in their matrix wave equation, which is actually a step backward with respect to our matrix wave equation.

III. CONCLUSION

The matrix wave equation (presented in our original article[2]) is not only applicable for wind and temperature effects, but for pressure variations as well. As long as the medium is stratified, any source or receiver height can be introduced. The improvements suggested by Raspet *et al.* are of the second order.

The theory is basically three dimensional but our computer model is restricted to two dimensions for practical reasons. This two-dimensional model is able to predict three-dimensional sound propagation from a point source if the medium is symmetric with respect to the plane $y = 0$, which is the case for temperature gradients and for wind gradients if the wind vector points in the x direction. For situations with a significant cross wind component a three-dimensional implementation of the method will be necessary for accurate predictions of the sound field. In the two-dimensional calculations, large vertical separations between source and receiver should be avoided.

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Comments on "The influence of wind and temperature gradients on sound propagation, calculated with the two way wave equations" [J. Acoust. Soc. Am. 87, 1987–1998 (1990)]

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In a recent paper Nijs and Wapenaar [J. Acoust. Am. **87**, 1987–1998 (1990)] have developed a method for solving for the effects of horizontal winds on sound propagation in the atmosphere. In this development, Nijs and Wapenaar neglect terms containing the density gradient of the atmosphere which in their notation are comparable to the effects of the adiabatic temperature lapse rate on the sound-speed gradient. Their method represents a significant improvement over the use of an effective sound-speed gradient to express the effects of the wind only for high propagation angles. For these situations, one would expect errors due to the change in ambient pressure and density to be greatest. In this letter, a numerically attractive method to incorporate horizontal winds and a hydrostatic density gradient into a two-dimensional transform solution for sound propagation in the atmosphere is developed. In addition, Nijs and Wapenaar approximate the two-dimensional Fourier transforms by a single one-dimensional transform with little justification. Conditions are discussed under which such an approximation may be valid and it is demonstrated that these conditions are unlikely to be met by the two-dimensional kernal of Nijs and Wapenaar.

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INTRODUCTION

Nijs and Wapenaar derived a two-dimensional Fourier transform solution for sound propagation in a layered atmosphere above a complex impedance ground surface.[1] Their solution assumes that the winds are horizontal and the properties of the media only vary with altitude. In addition, they have assumed that the ambient pressure and density do not vary with height. Nijs and Wapenaar recognize that this latter assumption may lead to inaccuracies in atmospheric predictions and show that the neglected density gradient terms are of the same order of magnitude as gradients due to an adiabatic lapse rate of temperature.

Previous fast Fourier transform methods[2] of predicting the effect of wind and temperature gradients on sound propagation have used the approximation that an effective sound speed in the direction of propagation can be used to describe the contribution of horizontal winds to refraction:

$$c_{\text{eff}} = c(z) + w(z)\cos[\theta(z)], \tag{1}$$

where $c(z)$ is the sound speed in still air at altitude z, $w(z)$ the wind speed, and $\theta(z)$ the angle between the direction the wind is blowing to at the altitude z and the horizontal component of propagation direction. This approximation is only useful for low vertical propagation angles. The fast-field methods incorporate the density variation with height into layers with constant density, constant sound velocity, wind speed, and wind direction. Gradients of these properties are modeled by the use of multiple thin layers.

Nijs and Wapenaar's method represents a significant improvement for sound propagation when the source and receiver have large vertical separation or the atmosphere is strongly refracting so that ray paths are at steep vertical angles. However, the variations with ambient pressure and density which the Nijs and Wapenaar method does not include will be greatest for these situations.

In addition, Nijs and Wapenaar assume that a one-dimensional Fourier transform will give correct results for propagation in the spatial direction corresponding to the wave-number component for propagation in the downwind or upwind. No direct justification is given for such an assumption.

In this letter, we drive the matrix equations in pressure and particle velocity to correctly describe sound propagation in an atmosphere with altitude dependent pressure, density, sound speed, and horizontal wind vector, and investigate the limitations imposed by the use of the one-dimensional Fourier transform in place of the two-dimensional transform.

I. THEORY

The matrix relations between pressure and particle velocity for a horizontally stratified media can be derived

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based on Pierce's derivation of a wave equation for fluids with unsteady inhomogeneous flow.[3] Pierce derives a linear wave equation correct to first order in the ratio of acoustic wavelength to inhomogeneity length scale and also to first order in ratio of wave period to inhomogeneity time scale. Consequently, these equations do not apply to very low-frequency propagation in the atmosphere.

We take Pierce's Eqs. (10a) and (10b) as our starting point:

$$D_t \mathbf{v}' + (\mathbf{v}'\cdot\nabla)\mathbf{v}_0 + \frac{1}{\rho_0}\nabla p' - \frac{p'}{(\rho_0 c)^2}\nabla p_0 + \left(\frac{\partial p}{\partial s}\right)_0 \frac{s'}{(\rho_0 c)^2}\nabla p_0 = 0, \quad (2)$$

$$D_t\left(\frac{p'}{\rho_0 c^2}\right) + \frac{1}{\rho_0 c^2}\mathbf{v}\cdot\nabla p_0 + \nabla\cdot\mathbf{v}' - s' D_t\left[\frac{1}{\rho_0 c^2}\left(\frac{\partial p}{\partial s}\right)_0\right] = 0, \quad (3)$$

where $D_t = \partial/\partial t + \mathbf{v}_0\cdot\nabla$; p', $\mathbf{v}'$, and s' are the field quantities of pressure, particle velocity, and entropy, c is the speed of sound, ρ_0 the ambient density, p_0 the ambient pressure and $\mathbf{v}_0$ the wind velocity. Pierce notes that s' is a first-order quantity in wavelength to inhomogeneity length scale and the last two terms in Eqs. (2) and (3) are therefore second order and may be omitted.

At this point, we diverge from Pierce and apply Nijs and Wapenaar's conditions to Eqs. (2) and (3); namely, the wind velocity is horizontal and all spatial variations of ambient conditions are functions only of altitude. To match Nijs and Wapenaar, we let $\mathbf{v}_0 = \mathbf{w}$, and $\hat{k}$ be the unit vector in the z direction. Equations (2) and (3) become

$$\nabla p' + \rho_0\left(\frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla\right)\mathbf{v}' + \rho_0 v_z' \frac{\partial \mathbf{w}}{\partial z} - \frac{\hat{k}p'}{\rho_0 c^2}\frac{\partial p_0}{\partial z} = 0; \quad (4)$$

$$\nabla\cdot\mathbf{v}' + \left(\frac{\partial}{\partial t} + \mathbf{w}\cdot\nabla\right)\frac{p'}{\rho_0 c^2} + \frac{1}{\rho_0 c^2} v_z' \frac{\partial p_0}{\partial z} = 0. \quad (5)$$

Comparison of these equations with Eqs. (5) and (6) of Nijs and Wapenaar shows that their equations do not contain the last terms in Eqs. (4) and (5).

Nijs and Wapenaar have restricted their analysis to an isobaric atmosphere, and so the matrix equations they derive are only correct for that case. It is not clear if the assumption of an isobaric atmosphere in their notation will produce realistic results for comparison with outdoor sound propagation data. The introduction of horizontal winds instead of an effective sound velocity into the calculations should only be significant for high angle propagation where the effects of wind are not equivalent to a change in sound velocity, but the ambient density and pressure variation then should affect the wave variables.

Equations (4) and (5) may be two dimensionally Fourier transformed and a propagation matrix similar to Eq. (17) of Ref. 1 developed for the vertical component of particle velocity and the pressure in an atmosphere with vertically varying ambient density, ambient pressure, and horizontal winds. In the following, we will use $e^{-i\omega t}$ notation, rather than Nijs and Wapenaar's $e^{+j\omega t}$ notation. The transform equations for pressure are

$$P'(k_x,k_y,z,\omega) = \int\int\int_{-\infty}^{\infty} p'(x,y,z,t)e^{i\omega t}e^{-ik_x x}e^{-ik_y y}dt\,dx\,dy \quad (6)$$

and

$$p'(x,y,z,t) = \frac{1}{8\pi^3}\int\int\int_{-\infty}^{\infty} P'(k_x,k_y,z,\omega) \times e^{-i\omega t}e^{ik_x x}e^{ik_y y}d\omega\,dk_x\,dk_y. \quad (7)$$

Using similar transform equations for the components of particle velocity, we find

$$-i\omega\rho_0\left(1 - \frac{k_x w_x}{\omega} - \frac{k_y w_y}{\omega}\right)V_x' + \rho_0 V_z' \frac{dw_x}{dz} + ik_x P' = 0; \quad (8)$$

$$-i\omega\rho_0\left(1 - \frac{k_x w_x}{\omega} - \frac{k_y w_y}{\omega}\right)V_y' + \rho_0 V_z' \frac{dw_y}{dz} + ik_y P' = 0; \quad (9)$$

$$-i\omega\rho_0\left(1 - \frac{k_x w_x}{\omega} - \frac{k_y w_y}{\omega}\right)V_z' + \frac{\partial P'}{\partial z} - \frac{P'}{\rho_0 c^2}\frac{dp_0}{dz} = 0; \quad (10)$$

and

$$\frac{-i\omega}{\rho_0 c^2}\left(1 - \frac{k_x w_x}{\omega} - \frac{k_y w_y}{\omega}\right)P' + \frac{V_z'}{\rho_0 c^2}\frac{dp_0}{dz} + ik_x V_x' + ik_y V_y' + \frac{\partial V_z'}{\partial z} = 0. \quad (11)$$

With the substitution $\mu = 1 - k_x w_x/c - k_y w_y/c$ and $k = \omega/c$ Eqs. (8), (9), and (10) can be reduced to equations relating to p' and V_z':

$$\frac{\partial P'}{\partial z} = \frac{1}{\rho_0 c^2}\frac{dp_0}{dz}P' + i\omega\rho_0\mu V_z', \quad (12)$$

and

$$\frac{\partial V_z'}{\partial z} = \frac{i}{\omega\rho_0\mu}[\mu^2 k^2 - k_x^2 - k_y^2]P' - \left[\frac{1}{\omega\mu}\left(k_x\frac{dw_x}{dz} + k_y\frac{dw_y}{dz}\right) + \frac{1}{\rho_0 c^2}\frac{dp_0}{dz}\right]V_z'. \quad (13)$$

These equations will reduce to a form solvable by the propagation matrix method for a model atmosphere composed of layers with constant temperature and constant wind speed under the condition of hydrostatic equilibrium. Within each layer p_0 and ρ_0 are exponentially decreasing with height as $\exp(-\gamma g z/c^2)$, where γ is the ratio of specific heats for atmosphere and g is the acceleration due to gravity. The effects of gradients of sound speed, wind speed, and wind direction on sound propagation may be modeled using multiple thin layers. The ambient pressure and density are related by $\gamma p_0/\rho_0 = c^2$, where c is a function only of temperature. Within each layer Eqs. (12) and (13) become

$$\frac{\partial P'}{\partial z} = -(g/c^2)P' + i\omega\rho_0\mu V_z', \tag{14}$$

and

$$\frac{\partial V_z'}{\partial z} = \frac{i}{\omega\rho_0\mu}(\mu^2k^2 - k_x^2 - k_y^2)P' + \frac{g}{c^2}V_z'. \tag{15}$$

With the definitions of new variables P'' and V_z'' such that $P' = P''\sqrt{\rho_0}$ and $V_z' = V_z''/\sqrt{\rho_0}$, Eqs. (14) and (15) become

$$\frac{\partial}{\partial z}\begin{bmatrix} P'' \\ V_z'' \end{bmatrix} = A\begin{bmatrix} P'' \\ V_z'' \end{bmatrix} + \begin{bmatrix} 0 \\ S\delta(z-z_s) \end{bmatrix}, \tag{16}$$

where

$$A_{11} = -(1-\gamma/2)g/c^2,$$

$$A_{12} = i\omega\mu,$$

$$A_{21} = (i/\omega\mu)(\mu^2k^2 - k_x^2 - k_y^2),$$

and

$$A_{22} = (1-\gamma/2)g/c^2.$$

The $A_{11}P''$ term and $A_{22}V_z''$ terms are on the order of $g/\omega c$ smaller than the $A_{12}V_z''$ and $A_{21}P''$ terms respectively for k_x and k_y small. The scaling of the pressure variable by the density will be significant for propagation from a source to a receiver with wide vertical separations.

Equation (16), along with the boundary conditions, can be used to solve for the Fourier transform of the pressure and vertical particle velocity. The time domain pressure and particle velocity can then be calculated with the inverse Fourier transform. In fast field programs, one Fourier transform calculates the pressure dependences with radial distance from the source. In the Nijs and Wapenaar notation, a two-dimensional discrete Fourier transform transform will produce the pressure as a function of x and y for fixed source and receiver heights.

II. NUMERICAL METHODS

Nijs and Wapenaar have restricted their results to the use of a one-dimensional Fourier transform and display results only for upwind and downwind propagation. A one-dimensional transform with k_x equivalent to a two-dimensional transform in k_x and k_y only if the kernal of the transform does not depend on k_y or has a delta function dependence on k_y.

Even if w_y is zero, the propagation matrix in Eq. (17) of Ref. 1 will contain k_y. That Nijs and Wapenaar obtain reasonable predictions from the one-dimensional transform indicates a delta function behavior of the kernal with k_y. A theoretical or numerical evaluation of the behavior of the kernal in two dimensions would serve to clarify this point. The matrix element A_{21} contains the same dependence on k_y as Eq. (17) of Ref. 1. If a two-dimensional transform is necessary for accurate prediction it will be computationally expensive, however the use of the fast Fourier transform in two dimensions will produce a map of sound-pressure level versus position for given source and receiver heights around a spherically symmetrical source in a horizontally stratified wind and temperature field.

III. CONCLUSIONS

Nijs and Wapenaar have addressed an important problem in outdoor sound propagation. The inclusion of the wind as a horizontal vector into an FFP like calculation is necessary for the prediction of sound propagation for nonhorizontal geometries typical of aircraft noise. However, their notation and results are limited to near horizontal propagation where the use of an effective sound speed is valid. That temperature gradients are equivalent to wind speed gradients for their test cases is noted in Sec. III A of Ref. 1.

We have derived a propagation matrix that includes the variation of pressure and density with altitude. A numerically attractive form of the propagation equations is developed for a model atmosphere composed of layers with constant temperature, constant wind speed, constant wind direction in which the ambient density obeys the hydrostatic equation. In the future, we plan to investigate the behavior of the two-dimensional kernal and to develop methods of evaluating the two dimensional sound pressure field using two-dimensional transforms.

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[3] Allan D. Pierce, "Wave equation for sound in fluids with unsteady inhomogeneous flow," J. Acoust. Soc. Am. 87, 2292–2299 (1990).