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Short Note

Reciprocity properties of one-way propagators

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INTRODUCTION

Acoustic reciprocity is a fundamental property of the wave equation for the total acoustic wavefield (Rayleigh, 1878). In its most elementary form the reciprocity principle states that an acoustic response remains the same when the source and receiver are interchanged. In terms of Green's functions this reciprocity principle reads $G(\mathbf{r}_2, \mathbf{r}_1) = G(\mathbf{r}_1, \mathbf{r}_2)$, with $\mathbf{r} = (x, y, z)$. For more general expressions and their seismic applications, see Fokkema and van den Berg (1993).

Evidently, a reciprocity principle for one-way propagators would imply that the propagators for downgoing waves are identical to those for upgoing waves [i.e., $\mathcal{W}^+(\mathbf{r}_2, \mathbf{r}_1) = \mathcal{W}^-(\mathbf{r}_1, \mathbf{r}_2)$]. However, since one-way propagators are solutions of the one-way wave equations, it is not at all obvious that they obey reciprocity. As a matter of fact, for the usual one-way propagators the reciprocity principle breaks down even in a simple laterally invariant two-layer configuration. This is because the transmission coefficients at an interface for downgoing and upgoing waves are not identical (they are given by $T^+ = 1 + R$ and $T^- = 1 - R$, respectively, where R is the reflection coefficient of the interface).

Whether the reciprocity principle for one-way propagators is fulfilled or not depends on how the total wavefield is decomposed into one-way wavefields. For horizontally layered media there is a vast amount of literature that makes use of the so-called flux-normalized decomposition (see, e.g., Frasier, 1970; Ursin, 1983; Burridge and Chang, 1989). Using flux normalization, the transmission coefficients at an interface for downgoing and upgoing waves are identical (they are both given by $\sqrt{1 - R^2}$) and, as a consequence, flux-normalized one-way propagators in laterally invariant media obey reciprocity.

The flux-normalized decomposition approach can be generalized for applications in arbitrarily inhomogeneous media (de Hoop, 1992, 1996). This generalization is based on a modified version of Claerbout's square-root operator (Wapenaar and Berkhout, 1989, App. B). In a recent paper we derived general reciprocity theorems for flux-normalized one-way wavefields and propagators in arbitrarily inhomogeneous media (Wapenaar and Grimbergen, 1996).

In this paper I review the different one-way propagators, show their mutual relation as well as their relation with the reciprocal Green's function, and discuss when and why the reciprocity principle is fulfilled or not. Understanding the reciprocity properties of (forward and inverse) one-way propagators is relevant for the design of true amplitude migration schemes.

PRESSURE-NORMALIZED DECOMPOSITION

Usually downgoing and upgoing waves P^+ and P^- are normalized such that their sum is equal (or proportional) to the acoustic pressure P of the total wavefield. Therefore, the usual decomposition will be referred to as pressure-normalized decomposition. In the seismic literature it was introduced by Claerbout (1971). Here I consider a slightly modified form of his relations.

In the space-frequency domain the relation between the acoustic pressure P and the vertical component of the particle velocity V_z on the one hand and the pressure-normalized downgoing and upgoing waves P^+ and P^- on the other hand reads

$$\begin{pmatrix} P \\ V_z \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\omega\varrho}\hat{H}_1 & \frac{-1}{\omega\varrho}\hat{H}_1 \end{pmatrix} \begin{pmatrix} P^+ \\ P^- \end{pmatrix}, \tag{1}$$

where ω is the angular frequency and ϱ the mass density. $\hat{H}_1$ is the square-root operator, which is related to the pseudo-Helmholtz operator $\hat{H}_2$ according to $\hat{H}_1\hat{H}_1 = \hat{H}_2$, with

$$\hat{H}_2 = \left(\frac{\omega}{c}\right)^2 + \varrho\frac{\partial}{\partial x}\left(\frac{1}{\varrho}\frac{\partial}{\partial x}\,\cdot\right) + \varrho\frac{\partial}{\partial y}\left(\frac{1}{\varrho}\frac{\partial}{\partial y}\,\cdot\right), \tag{2}$$

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where c is the propagation velocity. Inverting equation (1) yields the following decomposition relation:

$$\begin{pmatrix} P^+ \\ P^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & (\hat{H}_1^{-1} \omega \varrho \cdot) \\ 1 & -(\hat{H}_1^{-1} \omega \varrho \cdot) \end{pmatrix} \begin{pmatrix} P \\ V_z \end{pmatrix}. \qquad (3)$$

For laterally invariant media equations (1) and (3) may be reformulated in the wavenumber-frequency domain according to

$$\begin{pmatrix} \tilde{P} \\ \tilde{V}_z \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{k_z}{\omega \varrho} & -\frac{k_z}{\omega \varrho} \end{pmatrix} \begin{pmatrix} \tilde{P}^+ \\ \tilde{P}^- \end{pmatrix} \qquad (4)$$

and

$$\begin{pmatrix} \tilde{P}^+ \\ \tilde{P}^- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \frac{\omega \varrho}{k_z} \\ 1 & -\frac{\omega \varrho}{k_z} \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{V}_z \end{pmatrix}, \qquad (5)$$

where $k_z = \sqrt{\omega^2/c^2 - |\mathbf{k}|^2}$, with $\mathbf{k} = (k_x, k_y)$. From the latter equations the reflection and transmission coefficients at a horizontal interface between two homogeneous layers can be derived easily by using the continuity of $\tilde{P}$ and $\tilde{V}_z$ at the interface. The result is

$$\tilde{R} = \frac{\varrho_2/k_{z,2} - \varrho_1/k_{z,1}}{\varrho_2/k_{z,2} + \varrho_1/k_{z,1}} \qquad (6)$$

and

$$\tilde{T}^+ = 1 + \tilde{R}, \qquad \tilde{T}^- = 1 - \tilde{R}, \qquad (7)$$

where subscripts 1 and 2 refer to the two layers. Note that

$$\tilde{T}^+ = \frac{k_{z,1}}{k_{z,2}} \frac{\varrho_2}{\varrho_1} \tilde{T}^-. \qquad (8)$$

PRESSURE-NORMALIZED EXTRAPOLATION

Forward extrapolation

In the space-frequency domain, forward one-way wavefield extrapolation of a pressure-normalized downgoing wavefield P^+ from depth level z_1 to depth level z_2 (with $z_2 > z_1$) is formally described by

$$P^+(\mathbf{x}, z_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W^+(\mathbf{x}, z_2; \mathbf{x}', z_1) P^+(\mathbf{x}', z_1)\, d^2\mathbf{x}', \qquad (9)$$

where $\mathbf{x} = (x, y)$ (Schneider, 1978; Berkhout and van Wulfften Palthe, 1979). Similarly, for a pressure-normalized upgoing wave we may write

$$P^-(\mathbf{x}', z_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W^-(\mathbf{x}', z_1; \mathbf{x}, z_2) P^-(\mathbf{x}, z_2)\, d^2\mathbf{x}. \qquad (10)$$

Assuming an arbitrarily inhomogeneous medium in the region $z_1 < z < z_2$ and ignoring scattering contributions from regions $z \le z_1$ and $z \ge z_2$, the pressure-normalized one-way propagators $W^{\pm}$ may be expressed in terms of the Green's function according to

$$W^+(\mathbf{x}, z_2; \mathbf{x}', z_1) = \frac{\partial G(\mathbf{x}, z_2; \mathbf{x}', z_1)}{\partial z_1} \frac{2}{\varrho(\mathbf{x}', z_1)} \qquad (11)$$

and

$$W^-(\mathbf{x}', z_1; \mathbf{x}, z_2) = -\frac{\partial G(\mathbf{x}', z_1; \mathbf{x}, z_2)}{\partial z_2} \frac{2}{\varrho(\mathbf{x}, z_2)}. \qquad (12)$$

The Green's function G represents the response of a monopole source, observed by a monopole receiver; its reciprocity property reads $G(\mathbf{x}, z_2; \mathbf{x}', z_1) = G(\mathbf{x}', z_1; \mathbf{x}, z_2)$. According to equations (11) and (12) each of the one-way propagators represents the response of a scaled dipole source, observed by a monopole receiver. This asymmetry between the source and receiver characteristics implies that in general these pressure-normalized one-way propagators are not reciprocal [i.e., $W^+(\mathbf{x}, z_2; \mathbf{x}', z_1) \neq W^-(\mathbf{x}', z_1; \mathbf{x}, z_2)$; for the special case of a homogeneous medium these propagators are reciprocal].

It is illustrative to relate the asymmetry in equations (11) and (12) to the asymmetry between the transmission coefficients for downgoing and upgoing waves. For the situation of two homogeneous layers, separated by a horizontal interface, equations (11) and (12) may be reformulated in the wavenumber-frequency domain according to

$$\tilde{W}^+(\mathbf{k}, z_2; z_1) = \tilde{G}(\mathbf{k}, z_2; z_1) \frac{2jk_{z,1}}{\varrho_1} \qquad (13)$$

and

$$\tilde{W}^-(\mathbf{k}, z_1; z_2) = \tilde{G}(\mathbf{k}, z_1; z_2) \frac{2jk_{z,2}}{\varrho_2}. \qquad (14)$$

(Bear in mind that $z_2 > z_1$.) In this domain the reciprocity principle for the Green's function reads $\tilde{G}(\mathbf{k}, z_2; z_1) = \tilde{G}(\mathbf{k}, z_1; z_2)$ and thus

$$\tilde{W}^+(\mathbf{k}, z_2; z_1) = \frac{k_{z,1}}{k_{z,2}} \frac{\varrho_2}{\varrho_1} \tilde{W}^-(\mathbf{k}, z_1; z_2). \qquad (15)$$

Note the consistency with equation (8).

Inverse extrapolation

Inverse extrapolation of pressure-normalized one-way wavefields is formally described by

$$P^+(\mathbf{x}', z_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F^+(\mathbf{x}', z_1; \mathbf{x}, z_2) P^+(\mathbf{x}, z_2)\, d^2\mathbf{x} \qquad (16)$$

and

$$P^-(\mathbf{x}, z_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F^-(\mathbf{x}, z_2; \mathbf{x}', z_1) P^-(\mathbf{x}', z_1)\, d^2\mathbf{x}'. \qquad (17)$$

The inverse propagator for downgoing waves may be approximated by the complex conjugated forward propagator for upgoing waves and vice versa, according to (Wapenaar and Berkhout, 1989, chap. 7)

$$F^+(\mathbf{x}', z_1; \mathbf{x}, z_2) \approx [W^-(\mathbf{x}', z_1; \mathbf{x}, z_2)]^* \qquad (18)$$

and

$$F^{-}(\mathbf{x}, z_2; \mathbf{x}', z_1) \approx [W^{+}(\mathbf{x}, z_2; \mathbf{x}', z_1)]^{*}. \tag{19}$$

In general these propagators are not reciprocal [i.e., $F^{+}(\mathbf{x}', z_1; \mathbf{x}, z_2) \neq F^{-}(\mathbf{x}, z_2; \mathbf{x}', z_1)$]. The approximations in equations (18) and (19) involve the negligence of evanescent waves [even for homogeneous media (Berkhout and van Wulfften Palthe, 1979)] and the negligence of transmission losses and other second- and higher order scattering terms. The latter effect can be demonstrated easily at the hand of the two-layer configuration discussed earlier. In the wavenumber-frequency domain one obtains for propagating waves, i.e., for $|\mathbf{k}|^2 < \omega^2/c^2$,

$$\tilde{F}^{+}\tilde{W}^{+} \approx [\tilde{W}^{-}]^{*}\tilde{W}^{+} = \tilde{T}^{-}\tilde{T}^{+} = 1 - \tilde{R}^2. \tag{20}$$

For small contrasts, the second-order error term ($\tilde{R}^2$) is negligible. For complex media with significant scattering, the cumulative second- and higher order errors may become significant, so more advanced approaches are required. A further discussion is beyond the scope of this paper.

FLUX-NORMALIZED DECOMPOSITION

In essence, the absence of reciprocity for the pressure-normalized one-way propagators can be contributed to the asymmetry between the composition and decomposition relations (1) and (3). To obtain a more symmetric form, first the pseudo-Helmholtz operator $\hat{H}_2$, defined in equation (2), is reformulated according to

$$\hat{\mathcal{H}}_2 = \varrho^{-\frac{1}{2}}\left(\hat{H}_2 \varrho^{\frac{1}{2}} \cdot\right) \tag{21}$$

or, using equation (2),

$$\hat{\mathcal{H}}_2 = \left(\frac{\omega}{c'}\right)^2 + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{22}$$

where

$$\left(\frac{\omega}{c'}\right)^2 = \left(\frac{\omega}{c}\right)^2 - \frac{3\left(\left(\frac{\partial \varrho}{\partial x}\right)^2 + \left(\frac{\partial \varrho}{\partial y}\right)^2\right)}{4\varrho^2} + \frac{\frac{\partial^2 \varrho}{\partial x^2} + \frac{\partial^2 \varrho}{\partial y^2}}{2\varrho} \tag{23}$$

(modified after Brekhovskikh, 1960). Note that $\hat{\mathcal{H}}_2$, as defined in equation (22), is a true Helmholtz operator; equation (23) has the form of the Klein-Gordon dispersion relation, known from relativistic quantum mechanics and electromagnetic wave theory (Messiah, 1962; Anno et al., 1992). A modified square-root operator $\hat{\mathcal{H}}_1$ is introduced, according to $\hat{\mathcal{H}}_1\hat{\mathcal{H}}_1 = \hat{\mathcal{H}}_2$. De Hoop (1992, 1996) and Wapenaar and Grimbergen (1996) show that with this modified square-root operator the following symmetric composition and decomposition relations can be obtained:

$$\begin{pmatrix} P \\ V_z \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\omega\varrho)^{\frac{1}{2}}\hat{\mathcal{H}}_1^{-\frac{1}{2}} & (\omega\varrho)^{\frac{1}{2}}\hat{\mathcal{H}}_1^{-\frac{1}{2}} \\ (\omega\varrho)^{-\frac{1}{2}}\hat{\mathcal{H}}_1^{\frac{1}{2}} & -(\omega\varrho)^{-\frac{1}{2}}\hat{\mathcal{H}}_1^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \mathcal{P}^{+} \\ \mathcal{P}^{-} \end{pmatrix} \tag{24}$$

and

$$\begin{pmatrix} \mathcal{P}^{+} \\ \mathcal{P}^{-} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{\mathcal{H}}_1^{\frac{1}{2}}(\omega\varrho)^{-\frac{1}{2}} & \hat{\mathcal{H}}_1^{-\frac{1}{2}}(\omega\varrho)^{\frac{1}{2}} \\ \hat{\mathcal{H}}_1^{\frac{1}{2}}(\omega\varrho)^{-\frac{1}{2}} & -\hat{\mathcal{H}}_1^{-\frac{1}{2}}(\omega\varrho)^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} P \\ V_z \end{pmatrix}. \tag{25}$$

For laterally invariant media equations (24) and (25) may be reformulated in the wavenumber-frequency domain according to

$$\begin{pmatrix} \tilde{P} \\ \tilde{V}_z \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{\omega\varrho}{k_z}} & \sqrt{\frac{\omega\varrho}{k_z}} \\ \sqrt{\frac{k_z}{\omega\varrho}} & -\sqrt{\frac{k_z}{\omega\varrho}} \end{pmatrix} \begin{pmatrix} \tilde{\mathcal{P}}^{+} \\ \tilde{\mathcal{P}}^{-} \end{pmatrix} \tag{26}$$

and

$$\begin{pmatrix} \tilde{\mathcal{P}}^{+} \\ \tilde{\mathcal{P}}^{-} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{k_z}{\omega\varrho}} & \sqrt{\frac{\omega\varrho}{k_z}} \\ \sqrt{\frac{k_z}{\omega\varrho}} & -\sqrt{\frac{\omega\varrho}{k_z}} \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{V}_z \end{pmatrix} \tag{27}$$

(see, e.g., Ursin, 1983). From equation (27) one easily obtains for propagating waves, i.e., for $|\mathbf{k}|^2 < \omega^2/c^2$,

$$\tilde{\mathcal{P}}^{+}(\tilde{\mathcal{P}}^{+})^{*} - \tilde{\mathcal{P}}^{-}(\tilde{\mathcal{P}}^{-})^{*} = \tilde{P}\tilde{V}_z^{*} + \tilde{P}^{*}\tilde{V}_z. \tag{28}$$

Since the right-hand side of equation (28) is proportional to the power flux in the z-direction, $\mathcal{P}^{+}$ and $\mathcal{P}^{-}$ will be called flux-normalized one-way wavefields. From equations (26) and (27) the reflection and transmission coefficients at an interface between two homogeneous layers can again be derived by using the continuity of $\tilde{P}$ and $\tilde{V}_z$ at the interface. The result is

$$\tilde{\mathcal{T}}^{+} = \tilde{\mathcal{T}}^{-} = \sqrt{1 - \tilde{R}^2}, \tag{29}$$

with $\tilde{R}$ given by equation (6). Note that

$$\tilde{\mathcal{T}}^{+} = \sqrt{\frac{k_{z,2}}{k_{z,1}}\frac{\varrho_1}{\varrho_2}}\tilde{T}^{+}, \quad \tilde{\mathcal{T}}^{-} = \sqrt{\frac{k_{z,1}}{k_{z,2}}\frac{\varrho_2}{\varrho_1}}\tilde{T}^{-}, \tag{30}$$

with $\tilde{T}^{+}$ and $\tilde{T}^{-}$ defined in equation (7).

FLUX-NORMALIZED EXTRAPOLATION

Forward extrapolation

The formal expressions for forward one-way wavefield extrapolation of flux-normalized downgoing and upgoing wavefields are again given by equations (9) and (10), with $P^{\pm}$ and $W^{\pm}$ replaced by their flux-normalized counterparts $\mathcal{P}^{\pm}$ and $\mathcal{W}^{\pm}$, respectively. For these flux-normalized wavefields and propagators we derived general reciprocity theorems (Wapenaar and Grimbergen, 1996) and representations that apply to arbitrarily inhomogeneous media. One of the results, relevant to this paper, is the following reciprocity relation for the flux-normalized one-way propagators:

$$\mathcal{W}^{+}(\mathbf{x}, z_2; \mathbf{x}', z_1) = \mathcal{W}^{-}(\mathbf{x}', z_1; \mathbf{x}, z_2). \tag{31}$$

It is illustrative to consider again the special situation of two homogeneous layers, separated by a horizontal interface. For that situation we may write, analogous to equation (30),

$$\tilde{\mathcal{W}}^{+} = \sqrt{\frac{k_{z,2}}{k_{z,1}}\frac{\varrho_1}{\varrho_2}}\tilde{W}^{+}, \qquad \tilde{\mathcal{W}}^{-} = \sqrt{\frac{k_{z,1}}{k_{z,2}}\frac{\varrho_2}{\varrho_1}}\tilde{W}^{-} \tag{32}$$

or, upon substitution of equations (13) and (14),

$$\tilde{\mathcal{W}}^{+}(\mathbf{k}, z_2; z_1) = \sqrt{\frac{2jk_{z,2}}{\varrho_2}}\tilde{G}(\mathbf{k}, z_2; z_1)\sqrt{\frac{2jk_{z,1}}{\varrho_1}} \tag{33}$$

and

$$\tilde{\mathcal{W}}^{-}(\mathbf{k}, z_1; z_2) = \sqrt{\frac{2jk_{z,1}}{\varrho_1}}\tilde{G}(\mathbf{k}, z_1; z_2)\sqrt{\frac{2jk_{z,2}}{\varrho_2}}. \tag{34}$$

These expressions exhibit more symmetry than equations (13) and (14). In equations (13) and (14) the scalar multipliers transform the monopole source of the Green's function into a dipole source, and they leave the monopole receiver intact. On the other hand, the scalar multipliers in equations (33) and (34) transform the monopole source and receiver of the Green's function both in the same manner, so the reciprocity of the Green's function is preserved in the one-way propagators. The characteristics of the transformed source and receiver are between those of a monopole and a dipole. Finally, note that in a homogeneous medium the flux-normalized and pressure-normalized propagators are identical.

Inverse extrapolation

The formal expressions for inverse flux-normalized one-way wavefield extrapolation are again given by equations (16) and (17), with $P^{\pm}$ and $F^{\pm}$ replaced by their flux-normalized counterparts $\mathcal{P}^{\pm}$ and $\mathcal{F}^{\pm}$, respectively. Because of the reciprocity relation for the forward propagators [equation (31)], the inverse propagators may now be expressed as

$$\mathcal{F}^{+}(\mathbf{x}', z_1; \mathbf{x}, z_2) \approx [\mathcal{W}^{+}(\mathbf{x}, z_2; \mathbf{x}', z_1)]^{*} \tag{35}$$

and

$$\mathcal{F}^{-}(\mathbf{x}, z_2; \mathbf{x}', z_1) \approx [\mathcal{W}^{-}(\mathbf{x}', z_1; \mathbf{x}, z_2)]^{*}. \tag{36}$$

Note the subtle difference with equations (18) and (19). Clearly these flux-normalized inverse propagators obey reciprocity, i.e.,

$$\mathcal{F}^{+}(\mathbf{x}', z_1; \mathbf{x}, z_2) = \mathcal{F}^{-}(\mathbf{x}, z_2; \mathbf{x}', z_1). \tag{37}$$

The approximations in equations (35) and (36) are the same as those in equations (18) and (19). This is seen most easily for the two-layer configuration where, for propagating waves [compare with equation (20)],

$$\tilde{\mathcal{F}}^{+}\tilde{\mathcal{W}}^{+} \approx [\tilde{\mathcal{W}}^{+}]^{*}\tilde{\mathcal{W}}^{+} = [\tilde{\mathcal{T}}^{+}]^{2} = 1 - \tilde{R}^{2}. \tag{38}$$

Again, to account for the transmission losses, etc., in complex media with significant scattering, a more advanced approach is required. A further discussion is beyond the scope of this paper.

CONCLUSIONS AND DISCUSSION

The usual pressure-normalized one-way propagators may be seen as the response of a dipole source, observed by a monopole receiver. Since the source and receiver have different characteristics, these one-way propagators do not obey reciprocity. Flux-normalized one-way propagators, on the other hand, have interchangeable source and receiver characteristics and thus they obey reciprocity.

One-way propagators play a central role in seismic migration: they downward extrapolate the downgoing and upgoing wavefields from the surface into the subsurface. Usually the source function is treated as the downgoing wavefield and the measured data as the upgoing wavefield. As long as amplitudes are not important, this approach functions satisfactorily. For true amplitude migration, however, a proper decomposition into downgoing and upgoing waves prior to migration is required. The choice of the propagators (pressure- or flux-normalized) dictates which decomposition process should be used. If one decides to use reciprocal one-way propagators (for instance, for efficiency reasons), flux-normalized decomposition is a prerequisite.

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