



Green's function retrieval by cross-correlation in case of one-sided illumination

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[1] The cross-correlation of acoustic wave fields at two receivers yields the exact Green's function between these receivers, provided the receivers are surrounded by sources on a closed surface. In most practical situations the sources are located on an open surface and as a consequence the illumination of the receivers is one-sided. In this Letter we discuss the conditions for accurate Green's function retrieval for the situation of one-sided illumination. It appears that the Green's function retrieval method benefits from the fact that the earth is inhomogeneous, without relying on assumptions about disorder. **Citation:** Wapenaar, K. (2006), Green's function retrieval by cross-correlation in case of one-sided illumination, *Geophys. Res. Lett.*, 33, L19304, doi:10.1029/2006GL027747.

1. Introduction

[2] It has been shown by many researchers in geophysics, ultrasonics and underwater acoustics that the cross-correlation of acoustic wavefields recorded by two different receivers yields the response at one of the receiver positions as if there was a source at the other [Weaver and Lobkis, 2001; Campillo and Paul, 2003]. Various theories have been developed to explain this phenomenon, ranging from diffusion theory for enclosures [Weaver and Lobkis, 2001], multiple scattering theory and stationary-phase theory for random media [Malcolm et al., 2004; Snieder, 2004] and reciprocity theory for deterministic and random media [Wapenaar, 2004; Weaver and Lobkis, 2004; van Manen et al., 2005]. The principle of recovering an acoustic response by cross-correlation is often called Green's function retrieval; in the geophysical literature it is also known as seismic interferometry [Schuster et al., 2004; Wapenaar and Fokkema, 2006; Snieder et al., 2006].

[3] The derivation based on reciprocity theory yields an exact representation of the Green's function in an arbitrary inhomogeneous acoustic or elastic lossless medium. This representation applies to an open configuration; the two receivers are assumed to be surrounded by sources on an arbitrarily shaped closed surface, in general not coinciding with a physical boundary. When the sources emit one-by-one transient signals (as for example in exploration seismology), the Green's function between the two receivers is obtained by cross-correlating the individual responses and integrating the result along the sources. On the other hand, for simultaneously acting uncorrelated noise sources (as in passive seismology) the Green's function is obtained from a

single cross-correlation of the noise registrations (in both cases the end result is actually the Green's function convolved with the autocorrelation of the source signal). The reconstructed Green's function does not only contain the ballistic wave between the two receiver points but also the coda due to multiple scattering between the inhomogeneities of the medium.

[4] The condition of having sources on a closed surface is seldom fulfilled in practical situations. When the medium is partly bounded by a free surface, it is sufficient to have sources on an open, arbitrarily shaped surface that, together with the free surface, forms again a closed surface surrounding the two receivers, see Figure 1a. Since the acoustic pressure (or in the elastodynamic case the traction vector) vanishes at the free surface, the integral along the remaining part of the closed surface is sufficient to retrieve the exact Green's function. An alternative, more intuitive explanation is that the free surface acts as a mirror which obviates the need of having sources on a closed surface. The configuration of Figure 1a may represent the situation of passive seismology, in which receivers at or below the earth's free surface register the wave field emitted by natural sources in the subsurface. Hence, for this situation cross-correlation of passive measurements recovers the Green's reflection response of the subsurface, including the coda [Wapenaar, 2004]. A real data application is discussed by Draganov et al. [2006].

[5] In many other situations in which the source locations are restricted to an open surface, there is no free surface to form a closed surface surrounding the receivers, hence, the illumination is one-sided, see Figure 1b. This type of configuration occurs for example in exploration seismology, where the sources are located at the earth's surface only. For this configuration cross-correlation methods have been developed by Schuster et al. [2004] as a method for seismic imaging, by Bakulin and Calvert [2004] for Green's function retrieval and by Verschuur and Berkhout [2005] for transforming surface related multiples into primaries. Impressive results have been obtained by these authors for ballistic waves, but the coda due to internal multiple scattering was not considered. Snieder et al. [2006] analyze the cross-correlation method for exploration seismology in more detail and conclude that artifacts appear in the reconstructed Green's function as a result of the fact that the source locations do not constitute a closed surface.

[6] Figure 1b may alternatively be seen as a plan view of the configuration for retrieval of the surface wave Green's function. For example, Sabra et al. [2005] consider noise sources along the coast of Southern California. Cross-correlations between broadband seismic stations on land yield the surface wave Green's function between these stations. Due to the fact that the source locations do not

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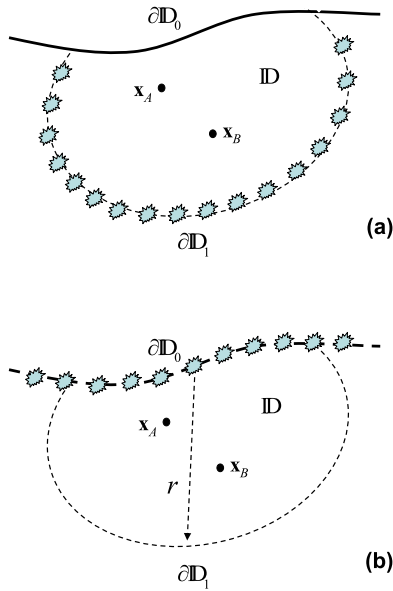


Figure 1. Exact Green's function retrieval by cross-correlation requires sources on a closed surface. (a) When part of the closed surface ($\partial\mathbb{D}_0$) is a free surface it suffices to have sources on the remaining part ($\partial\mathbb{D}_1$) of the closed surface. (b) In case of one-sided illumination (sources on $\partial\mathbb{D}_0$ only), the contribution of the remaining part of the closed surface ($\partial\mathbb{D}_1$) is ignored, which generally leads to artifacts. In this Letter we introduce an extinction condition for the integral over $\partial\mathbb{D}_1$.

form a closed contour surrounding the stations, their correlation results are asymmetric in time, which implies an incomplete reconstruction of the Green's function.

[7] In this Letter we briefly review the exact Green's function representation for the situation of sources on a closed surface, discuss the effect of one-sided illumination and introduce an extinction condition for the missing part of the integral. We show that under specific conditions the full Green's function (including the coda) can be retrieved from cross-correlations, even when the illumination is one-sided.

2. Green's Function Representation

[8] We consider a lossless arbitrary inhomogeneous anisotropic solid medium in which we define a domain \mathbb{D} enclosed by an arbitrarily shaped surface $\partial\mathbb{D}$ with outward pointing normal vector $\mathbf{n} = (n_1, n_2, n_3)$. In general the surface $\partial\mathbb{D}$ does not coincide with a physical boundary. Inside this domain we define two points \mathbf{x}_A and \mathbf{x}_B . In the frequency domain, the elastodynamic Green's function between these two points can be represented as [Wapenaar and Fokkema, 2006]

$$2\Re\left\{\hat{G}_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, \omega)\right\} = - \oint_{\partial\mathbb{D}} \left(\hat{G}_{q,ij}^{v,h}(\mathbf{x}_B, \mathbf{x}, \omega) \left\{ \hat{G}_{p,i}^{v,f}(\mathbf{x}_A, \mathbf{x}, \omega) \right\}^* + \hat{G}_{q,i}^{v,f}(\mathbf{x}_B, \mathbf{x}, \omega) \left\{ \hat{G}_{p,ij}^{v,h}(\mathbf{x}_A, \mathbf{x}, \omega) \right\}^* \right) n_j d^2\mathbf{x}, \quad (1)$$

where the asterisk denotes complex conjugation and \Re that the real part is taken. The notation convention for the

elastodynamic Green's function is as follows. The two coordinate vectors between the brackets represent the observation point and the source point, respectively. The superscripts represent the observed quantity and the source quantity, respectively (v standing for particle velocity, τ for stress, f for force and h for deformation). The subscripts represent the components of the observed quantity and the source quantity, respectively. Lowercase Latin subscripts take on the values 1, 2 and 3; Einstein's summation convention applies to repeated subscripts. The circumflex denotes that these Green's functions are represented in the frequency domain; ω denotes the angular frequency. Note that when the observed quantity would be displacement instead of velocity, then $2\Re$ in the left-hand side would be replaced by $2j\Im$ (where j denotes the imaginary unit and \Im the imaginary part), and the minus sign in the right-hand side by $j\omega$, corresponding to differentiation in the time domain.

[9] The products $\hat{G}_{q,ij}^{v,h} \{ \hat{G}_{p,i}^{v,f} \}^*$ etc. in equation (1) correspond to cross-correlations in the time domain. Hence, the right-hand side can be interpreted as the integral of the Fourier transform of cross-correlations of observed particle velocities at \mathbf{x}_A and \mathbf{x}_B , respectively, due to impulsive sources at \mathbf{x} on $\partial\mathbb{D}$; the integration takes place along the source coordinate \mathbf{x} . Since by reciprocity $\hat{G}_{ij,q}^{\tau,f}(\mathbf{x}, \mathbf{x}_B, \omega) n_j = \hat{G}_{ij,q}^{v,f}(\mathbf{x}, \mathbf{x}_B, \omega) n_j$ etc., the integrand vanishes at those parts of $\partial\mathbb{D}$ that coincide with a free surface. The left-hand side of equation (1) is the Fourier transform of $G_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, t) + G_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, -t)$, which is the superposition of the observed particle velocity in the x_q -direction at \mathbf{x}_B due to an impulsive force in the x_p -direction at \mathbf{x}_A and its time-reversed version. The Green's function $G_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, t)$ is obtained by taking the causal part of this superposition. Note that equation (1) is exact and applies to any lossless arbitrary inhomogeneous anisotropic solid medium. In practice data are band-limited, which implies that \mathbf{x}_A and \mathbf{x}_B should be sufficiently far apart to be outside the diffraction limit [van Manen et al., 2005]. The choice of the integration boundary $\partial\mathbb{D}$ is arbitrary (as long as it encloses \mathbf{x}_A and \mathbf{x}_B) and the medium may be inhomogeneous inside as well as outside $\partial\mathbb{D}$. The reconstructed Green's function contains, apart from the ballistic wave between \mathbf{x}_A and \mathbf{x}_B , all scattering contributions from inhomogeneities inside as well as outside $\partial\mathbb{D}$.

[10] When the medium outside $\partial\mathbb{D}$ is homogeneous and isotropic, with P - and S -wave propagation velocities c_P and c_S , respectively, and mass density ρ , equation (1) can be approximated by [Wapenaar and Fokkema, 2006]

$$2\Re\left\{\hat{G}_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, \omega)\right\} \approx \frac{2}{\rho c^K} \oint_{\partial\mathbb{D}} \hat{G}_{q,K}^{v,\phi}(\mathbf{x}_B, \mathbf{x}, \omega) \cdot \left\{ \hat{G}_{p,K}^{v,\phi}(\mathbf{x}_A, \mathbf{x}, \omega) \right\}^* d^2\mathbf{x}. \quad (2)$$

[11] Upper-case Latin subscripts take on the values 0, 1, 2 and 3; the repeated subscript K represents a summation from 0 to 3. In equation (2), $c^K = c_P$ for $K = 0$ and $c^K = c_S$ for $K = 1, 2, 3$. The Green's functions in the right-hand side represent again the observed particle velocities at \mathbf{x}_A and \mathbf{x}_B due to impulsive sources at \mathbf{x} on $\partial\mathbb{D}$. The superscript ϕ denotes that these sources are P -wave sources for $K = 0$, and S -wave sources with different polarizations for $K = 1, 2, 3$. Hence, the summation over the repeated subscript K repre-

sents a summation over P - and S -wave sources. For the acoustic situation K takes on the value 0 only (hence $c^K = c_P$) and the summation can be skipped. Since equation (2) contains only one correlation product, it is better suited for application in seismic interferometry than equation (1). In case of uncorrelated stationary noise sources equally distributed over $\partial\mathbb{D}$, whose spectra satisfy $\langle \hat{N}_K(\mathbf{x}, \omega) \hat{N}_L^*(\mathbf{x}', \omega) \rangle = \frac{2}{\rho c^K} \delta_{KL} \delta(\mathbf{x} - \mathbf{x}') \hat{S}(\omega)$ (where $\hat{S}(\omega)$ is the power spectrum of the noise), the right-hand side reduces to a direct cross-correlation of the observations at \mathbf{x}_A and \mathbf{x}_B , without the integral over $\partial\mathbb{D}$.

[12] For vertically propagating plane waves in a 1-D inhomogeneous medium, equation (2) simplifies to

$$2\Re\{\hat{G}(z_B, z_A, \omega)\} = \hat{G}(z_B, z_0, \omega)\{\hat{G}(z_A, z_0, \omega)\}^* + \hat{G}(z_B, z_1, \omega)\{\hat{G}(z_A, z_1, \omega)\}^*, \quad (3)$$

where z denotes depth. The two source positions z_0 and z_1 (with $z_0 < z_1$) enclose the 1-D inhomogeneous medium; the receiver positions z_A and z_B are situated somewhere between z_0 and z_1 . The plane wave Green's functions \hat{G} in equation (3) are flux-normalized, which explains the absence of the normalization factor $2/\rho c^K$. Since for the considered situation no wave conversion can take place, subscripts and superscripts have been omitted and equation (3) applies independently for P -waves as well as for S -waves.

3. Effect of One-Sided Illumination

[13] Evaluation of either equations (1) or (2) requires that sources are available on a closed surface $\partial\mathbb{D}$ around the observation points \mathbf{x}_A and \mathbf{x}_B . In this section we analyze the effect of one-sided illumination, as pictured in Figure 1b. Hence, we assume that the closed surface $\partial\mathbb{D}$ consists of a part $\partial\mathbb{D}_0$ containing sources and a part $\partial\mathbb{D}_1$ without sources. The integrals in equations (1) and (2) can now only be evaluated over $\partial\mathbb{D}_0$. We assume that $\partial\mathbb{D}_0$ is not a free surface, otherwise there would be no integral left to be evaluated.

[14] First we consider the application in exploration seismology. We consider responses from sources at the acquisition surface $\partial\mathbb{D}_0$, from which the free surface effects have been eliminated [Verschuur *et al.*, 1992]. Assuming the responses of these sources are measured by receivers at \mathbf{x}_A and \mathbf{x}_B in the subsurface (for example in a vertical seismic profile (VSP), a vertical array, a horizontal well, or at the ocean bottom), cross-correlation and integration along the sources on $\partial\mathbb{D}_0$ yields a non-exact reconstruction of the Green's function $G_{q,p}^{v,f}(\mathbf{x}_B, \mathbf{x}_A, \omega)$. Let us have a closer look at the neglected integral over $\partial\mathbb{D}_1$. Let $\partial\mathbb{D}_1$ be a half-sphere with radius r . If we take $r \rightarrow \infty$ and assume that the medium is homogeneous and isotropic outside a half-sphere with a fixed finite radius, then the P - and S -wave contributions of the Green's functions under the integral are $O(1/r)$ and each of the products is $O(1/r^2)$. Suppose we would consider the right-hand side of equation (1) without the complex conjugation signs and with the plus-sign replaced by a minus-sign, as in the well-known Rayleigh-Betti integral. In that case all terms of $O(1/r^2)$ would cancel each other, making the integrand $O(1/r^3)$ [Pao and Varatharajulu, 1976]. However, in equation (1) in its present form this cancellation does not take place, which means that the integrand is $O(1/r^2)$. Since the surface area of the integra-

tion boundary $\partial\mathbb{D}_1$ increases with r^2 , the integral over $\partial\mathbb{D}_1$ in equation (1) (and also in equation (2)) is $O(1)$. In other words, the boundary integral over $\partial\mathbb{D}_1$ does not vanish when $r \rightarrow \infty$. Since the integral in equation (1) is independent of the shape of $\partial\mathbb{D}$ (as long as it encloses \mathbf{x}_A and \mathbf{x}_B), the contribution of $\partial\mathbb{D}_1$ also does not vanish when r is finite.

[15] Next consider the situation for retrieval of the surface wave Green's function. Let $\partial\mathbb{D}_1$ in Figure 1b (which is now a plan view) be a cylindrical strip directly below the surface with radius $r \rightarrow \infty$. Assuming again that the medium is homogeneous and isotropic beyond some finite radius, the surface wave contributions of the Green's functions under the integral are $O(1/\sqrt{r})$ and each of the products is $O(1/r)$. No cancellation of the terms of $O(1/r)$ takes place, hence, since the perimeter of $\partial\mathbb{D}_1$ increases linearly with r , the neglected integral over $\partial\mathbb{D}_1$ in equations (1) and (2) is again $O(1)$.

[16] Finally consider the retrieval of the plane wave Green's function in a 1-D inhomogeneous medium, as formulated by equation (3). In case of one-sided illumination by a plane wave source at z_0 , the second term on the right-hand side of equation (3) cannot be evaluated. If we move z_1 away from the inhomogeneous medium, then the plane wave Green's functions are $O(1)$. Since no integration takes place, the neglected second term in equation (3), containing the product of Green's functions, is $O(1)$ as well.

[17] Summarizing, in case of one-sided illumination (sources at $\partial\mathbb{D}_0$ only, or in the 1-D case at z_0 only), the Green's function retrieved by cross-correlation contains a non-vanishing error. In general this implies that not only the amplitudes of the ballistic wave may be erroneously reconstructed, but also that multiply scattered events in the coda are incorrectly handled and that spurious events may occur. The occurrence of spurious events is discussed for the 3-D situation by Snieder *et al.* [2006]. Here we illustrate it with a simple plane-wave experiment for a 1-D inhomogeneous medium.

[18] Consider a horizontally layered medium, consisting of 25 layers with a thickness of 20 m each, with random P -wave velocities around an average velocity of 2000 m/s (Figure 2a). A vertically downward travelling plane P -wave is incident to this configuration at $z_0 = 0$ m. We consider receivers at $z_A = 100$ m and $z_B = 300$ m (as in a vertical seismic profile). The responses at z_A and z_B are shown in Figures 2b and 2c. We use the first term in the right-hand side of equation (3) to approximate the Green's function between z_A and z_B . In the time domain this comes to

$$G(z_B, z_A, -t) + G(z_B, z_A, t) \approx \int_{-\infty}^{\infty} G(z_B, z_0, t + t') G(z_A, z_0, t') dt'. \quad (4)$$

[19] The correlation result is shown in Figure 2d, the exact result in Figure 2e and a comparison between the two in Figure 2f. Note the asymmetry and the occurrence of spurious events around $t = 0$ in the correlation result due to the fact that no source was present at z_1 .

4. Extinction Condition for the Missing Integral

[20] Consider again the situation of one-sided illumination, as pictured in Figure 1b, with sources on $\partial\mathbb{D}_0$ only. In

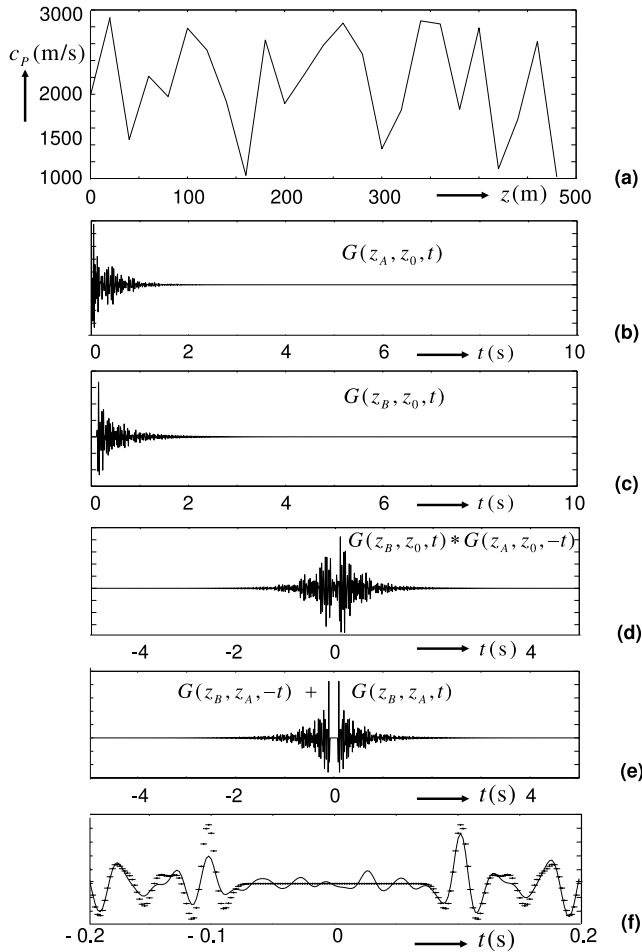


Figure 2. (a) Velocity profile (25 layers of 20 m each). (b) Plane wave response at $z_A = 100$ m. (c) Plane wave response at $z_B = 300$ m. (d) Cross-correlation result (the asterisk denotes convolution). (e) Directly modelled Green's function. (f) Comparison of Figures 2d (solid) and 2e (+).

the previous section we argued that the integral over $\partial\mathbb{D}_1$ does not vanish when $r \rightarrow \infty$. The decay of the integrand when r increases is precisely counterbalanced by the growth of the surface area of $\partial\mathbb{D}_1$. In the analysis we assumed that the medium is homogeneous and isotropic outside a half-sphere (or semi-circle) with a fixed finite radius, which is a quite common assumption in the analysis of integrals in unbounded configurations.

[21] Next we consider the situation in which the medium is inhomogeneous throughout the half-space bounded by $\partial\mathbb{D}_0$. A wave propagating through an inhomogeneous medium undergoes 'scattering loss' (which is not a real loss; the energy is just redistributed over the forward and backward scattered waves). Intuitively one might expect that the aforementioned balance would be disturbed due to the scattering loss. However, as already remarked in the discussion below equation (1), the result of the integral is independent of the choice of the integration boundary, even when the medium is inhomogeneous inside as well as outside this boundary. In order to analyze the effect of scattering losses on the integral over $\partial\mathbb{D}_1$, we follow a different approach. Let $\partial\mathbb{D}_1$ again be a half-sphere (or a

cylindrical strip for the surface wave analysis) with radius r . We consider the artificial situation, in which the medium is inhomogeneous throughout \mathbb{D} (i.e., the domain enclosed by $\partial\mathbb{D}_0 \cup \partial\mathbb{D}_1$), whereas it is homogeneous and isotropic outside \mathbb{D} . When r increases, the inhomogeneous domain increases as well, unlike in the analysis in the previous section, where the inhomogeneous domain was fixed. Due to internal scattering the decay of the integrand for increasing r is not fully compensated by the growth of the surface area of $\partial\mathbb{D}_1$. The integral over $\partial\mathbb{D}_1$ for large r is $O(f(r))$, where $f(r)$ is a decaying function accounting for the scattering losses. The behavior of $f(r)$ depends largely on the type and distribution of the inhomogeneities, which will not be discussed here. What matters is that it can become arbitrary small for large enough r and 'sufficient inhomogeneity' of the medium enclosed by $\partial\mathbb{D}_0 \cup \partial\mathbb{D}_1$.

[22] Hence, for the situation of one-sided illumination, the Green's function $\hat{G}_{q,p}^{w,f}(\mathbf{x}_B, \mathbf{x}_A, \omega)$ can be retrieved with arbitrary accuracy from the integral in equation (1) with closed surface $\partial\mathbb{D}$ replaced by open surface $\partial\mathbb{D}_0$ (see Figure 1b), by considering a large enough, sufficient inhomogeneous domain \mathbb{D} (and equation (2) gives a good approximation for the same situation). Similarly, in equation (3) the second term on the right-hand side is also $O(f(r))$ for large $r = z_1 - z_0$ when the medium is inhomogeneous in the entire region between z_0 and z_1 (assuming z_0 fixed and z_1 variable). Here $f(r)$ is again some decaying function which can become arbitrary small. Hence, for one-sided illumination the plane wave Green's function $\hat{G}(z_B, z_A, \omega)$ can be obtained with arbitrary accuracy from the first term only in the right-hand side of equation (3), assuming large enough $r = z_1 - z_0$ and sufficient inhomogeneity in the region between z_0 and z_1 .

[23] In both cases we see that the inhomogeneity of the medium helps to improve the Green's function retrieval when the illumination is one-sided. In some of the references mentioned in the introduction, the retrieval of the Green's function depends on the diffusivity of the wave field caused for example by multiple scattering in a random medium. In contrast, the extinction condition discussed above applies to a fully deterministic medium. The intuitive explanation why in this situation one-sided illumination suffices, is that the inhomogeneous medium acts as a 'mirror' (with a very complex phase behavior) for the sources at $\partial\mathbb{D}_0$ (Figure 1b), similar as the free surface acts as a mirror for the sources at $\partial\mathbb{D}_1$ in the case of passive seismology (Figure 1a). Since we assumed from the beginning that the medium is lossless, all energy emitted by the sources at $\partial\mathbb{D}_0$ is eventually reflected by this complex mirror, which compensates for the missing sources at $\partial\mathbb{D}_1$ in Figure 1b. We illustrate this theory with a plane wave experiment.

[24] We extend the horizontally layered medium of the previous example to a total of 250 layers with a thickness of 20 m each (Figure 3a). Again a vertically downward travelling plane P -wave is incident to this configuration at $z_0 = 0$ m. Figure 3b shows the transmission response as a function of r (for a given r this is the response of the medium of Figure 3a between z_0 and $z_0 + r$, embedded between homogeneous half-spaces). Figure 3c shows the total energy $f(r)$ of each trace of Figure 3b; note that this function decays monotonically. The responses at $z_A = 100$ m and $z_B = 300$ m of the entire medium of Figure 3a are shown

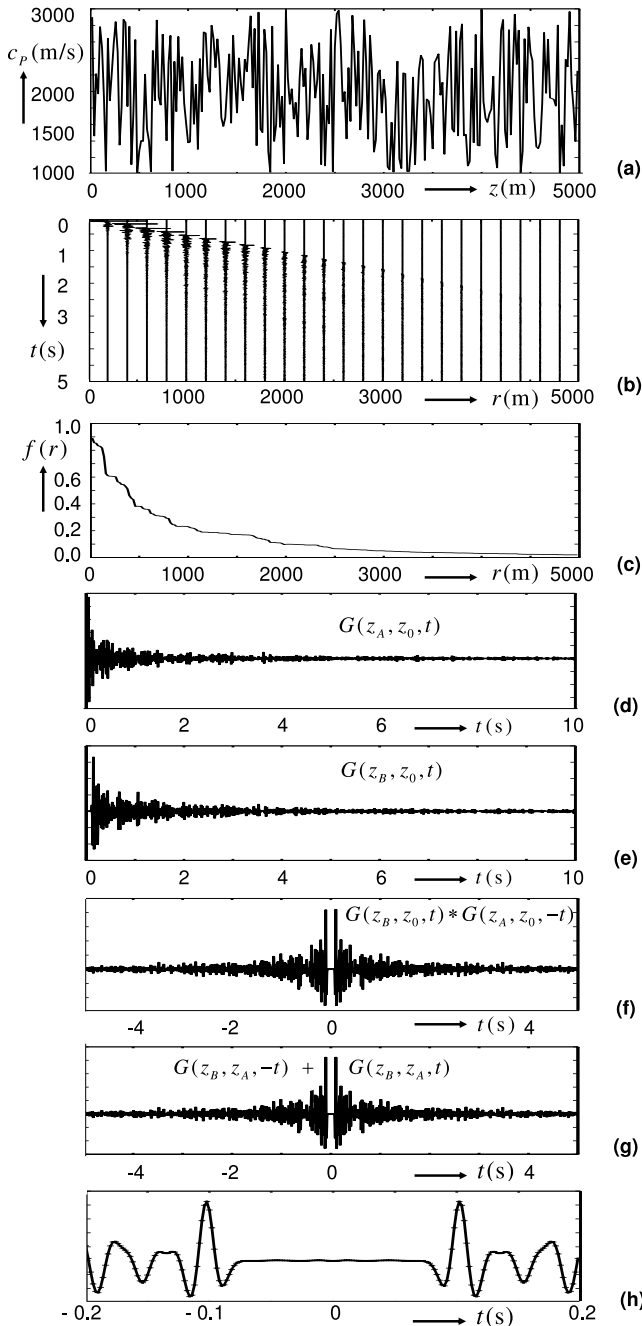


Figure 3. (a) Velocity profile (250 layers of 20 m each). (b) Flux-normalized transmission response, as a function of the total depth range of the medium. (c) The total energy $f(r)$ of each trace of Figure 3b. (d) Plane wave response at $z_A = 100$ m. (e) Plane wave response at $z_B = 300$ m. (f) Cross-correlation result. (g) Directly modelled Green's function. (h) Comparison of Figures 3f (solid) and 3g (+).

in Figures 3d and 3e. The main difference with the responses in Figures 2b and 2c is the longer coda. The cross-correlation result (equation (4)) is shown in Figure 3f, the exact result in Figure 3g and a comparison between the two in Figure 3h. Note that the correlation result is perfectly symmetric and that the spurious events around $t = 0$ have disappeared. Apparently the cross-correlation of the long

codas has contributed to the improved reconstruction of the Green's function and to the suppression of spurious events at early times.

5. Concluding Remarks

[25] From the theory discussed in this Letter as well as from the numerical examples it follows that Green's function retrieval in case of one-sided illumination (Figure 1b) benefits from the fact that the earth is inhomogeneous. Errors that would occur in the reconstructed Green's function when the response of only a few scatterers would be available are suppressed by cross-correlating the full response of the inhomogeneous medium. The reconstruction of the Green's function is the result of a complex interference of cross-correlated primaries and multiply scattered events, present in the coda of the response. It has been observed before that coda waves are surprisingly stable [Fink, 1997; Snieder and Scales, 1998], hence, we expect that this is not a limiting factor for practical applications. Note that, despite the complexity of the coda, this reconstruction process is fully deterministic and thus does not rely on diffusivity assumptions, unlike some of the references mentioned in the introduction.

[26] The theory discussed in this Letter applies to the situation of sources emitting transient signals one-by-one (as in exploration seismology), as well as for simultaneously acting uncorrelated noise sources (as for example in passive surface wave seismology). Aspects that may limit the accuracy of the retrieved Green's function in practice are anelastic losses, a finite source surface $\partial\mathbb{D}_0$, finite registration times and, for the situation of natural noise sources, mutual correlation and irregular source distribution. Investigations by Slob *et al.* [2006] for electromagnetic data indicate that when the losses are small, the cross-correlation method yields Green's functions with correct traveltimes and approximate amplitudes. It remains to be investigated how anelastic losses and other practical limitations will degrade the Green's function reconstruction for the situation of one-sided illumination.

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